Direct Link Aware Cooperative Relaying

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***Abstract*— Cooperative relaying strategies enable spatial diversity gains. Using a proper forwarding strategy, these techniques can achieve diversity order as high as the number of diverse paths. We derive a strategy named Direct Link-Aware Relaying (DLA), which uses the source-destination link status as a condition to forward or not the signal at the relay. DLA is shown to achieve full diversity and to save energy compared to decode-and-forward strategies. Compared to selective-forwarding, DLA overcomes the use of error detection codes. Extension to the Multi-Branch case is developed. Simulations corroborate our analytical claims.**

I. Introduction

As a result of the increasing development of wireless networks, the possibility of taking advantage of the distributed nature of this networks has aroused a great interest [1] [2][5] [6][7]. The capability of enabling spatial diversity in the same means as multi-input multi-output (MIMO) systems without using spatially separated antennas, can be achieved in these networks by using the idle nodes close to the transmitter and the receiver. This idle nodes enable different paths between the source node and the destination node, acting as relay nodes. If the information provided by the different paths can be properly mixed with the information transmitted by the source, diversity order proportional to the number of relays can be achieved. The greatest benefit of this cooperation is that no extra antenna is needed, so it can be implemented in small nodes.

Three main strategies have been described which follows this key idea of cooperative relaying. If the relay can amplify-and-forward (AF) the analog waveform from the source, an optimum combination of both direct and relayed signals can be accomplished at the receiver [5]. The main drawback of AF is that it is needed to mitigate the RF coupling effects. Another strategy solves this problem by decoding and storing the information before retransmitting it, which is called decode-and-forward (DF) [4]. Simple DF does not achieve full diversity gain by itself. Lastly, if the relay only transmits the correctly decoded messages, the implementation is called selective-relaying (SF) [4]. The main limitation of this protocol is that an ideal cyclic redundancy check (CRC) code is needed. This paper describes a SF-based strategy that achieves full diversity order in general scenarios.

Recently, three strategies derived from those ones have been shown to improve the performance of DF. In [6] a suboptimal detector called λ-MRC is presented, where the weight used for combine the instantaneous channel states is not analytically specified. This gap is covered by [8], which exploit knowledge of the instantaneous bit error probability (BEP) of the source-relay link at the destination, achieving full diversity gains for general scenarios and modulation schemes. Finally, in [9] an Adaptive Link Regenerative (LAR) strategy is presented. This technique weights the transmitted power at the relay with a factor between zero and one that depends on the state of the channels between the source and the relay and between the relay and the destination. This way, similar results as in [8] are obtained, but with significant energy savings at the relay.

In this paper we exploit the knowledge of the instantaneous BEP of the source-destination channel, in order to get a proper condition to relay or not the information. In other words, it is derived an on-off SF-based strategy in which the relay retransmits only when his cooperation is needed, depending on both the state of the direct channel and the state of the source relay channel. This way, where the direct signal is good enough or the relayed signal is expected to be unreliable, the relay remains quiet and saves power. Thus, the receiver implements two different algorithms for decoding the signals. On one hand, when the relay has retransmitted, it uses a conventional maximum ratio combiner (MRC) with both waveforms. On the other hand, when the relay has not retransmitted, the destination can only use the information from the source, and applies a maximum-likelihood detector (ML).

The performance of this strategy is tested via simulation, comparing the BEP provided by DLA with the one provided by the simple-input simple-output (SISO) system. It turns out that full diversity is achieved for general scenarios. Moreover, energy savings are also validated by comparison with LAR protocol. It is shown that some power gain is obtained for the same BEP. Thus, the relay can save more energy if DLA is used.

The remains of the paper is organized as follows. In Section II the system model and basic assumptions are presented and the notation is introduced. In Section III the protocol is described and in Section IV the mathematical analysis of its performance is carried out. Simulations and its results are described in Section V. Finally, Section VI sums up the main points of the paper.

II. System Model and Basic Assumptions

Let us consider the simplest model with one relay (R), which can be seen in Fig. 1. Let x be the signal to be transmitted from the source (S) to the destination (D) over the direct channel, called here source-destination channel ($S-D$).

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x is taken from a constellation $\mathcal{A}_x$. S broadcast x to D, and thus R receives the symbols too, over the source-relay channel $(S-R)$. Then, R can forward the signal to the D through the relay-destination channel $(R-D)$.

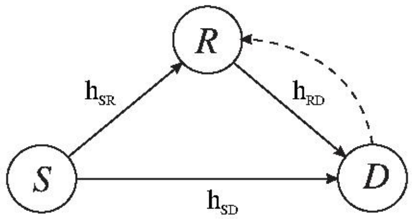

Fig. 1. Model for single relay system.

As wireless links are assumed, R and S must transmit their signals over orthogonal channels, and thus some kind of multiplexing is needed. Hereafter, we assume this is accomplished by a time division duplex mode, where time is divided in slots that are successively used by the different nodes. The channels between nodes are considered to be independent, and to be double-selective fading channels. Under both circumstances, a detector capable of collecting diversity can be constructed.

In the protocol presented here, the relay needs information about the direct link channel state. For this to be realistic, it is assumed that a low capacity feedback channel allows the destination to transmit the state of the source-destination channel to the relay.

In the first time slot, S broadcast the modulated symbol x. Received signals in R and D are

$$y_{sr} = h_{sr}x + z_{sr}, \quad (1)$$

$$y_{sd} = h_{sd}x + z_{sd}, \quad (2)$$

where z_{sd} and z_{sr} are the noise terms, with variances N_0, modelled as complex gaussian noise, i.e. $z_{SD} \sim \mathcal{CN}(0, N_0)$, $z_{sr} \sim \mathcal{CN}(0, N_0)$. Channel coefficients h_{sd}, h_{sr} are modelled as $h_{sd} \sim \mathcal{CN}(0, \sigma_{sd}^2)$ and $h_{sr} \sim \mathcal{CN}(0, \sigma_{sr}^2)$, with $\sigma_{sd} = E\{|h_{sd}|^2\}$ and $\sigma_{sr} = E\{|h_{sr}|^2\}$.

In the second slot, R transmits the estimated symbol to the destination,

$$y_{rd} = h_{rd}\hat{x} + z_{rd}, \quad (3)$$

where $\hat{x}$ is the estimated symbol at the receiver of the relay and all the other terms have analogous meaning as above, that is, $z_{RD} \sim \mathcal{CN}(0, N_0)$, $h_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$ and $\sigma_{rd} = E\{|h_{rd}|^2\}$.

For convenience, we define here the instantaneous signal-to-noise ratio (SNR) of each link as $\gamma_{sd} = |h_{sd}|^2\bar{\gamma}$, $\gamma_{sr} = |h_{sr}|^2\bar{\gamma}$ and $\gamma_{rd} = |h_{rd}|^2\bar{\gamma}$ with $\bar{\gamma} = P_x/N_0$ where P_x denotes the transmitted power.

III. Direct Link Aware Relaying

In non-selective forwarding strategies, the relay retransmits all the symbols it receives, even when they are not correct, and hence it wastes power. In selective forwarding strategies, an ideal CRC code is assumed. To overcome this limitation, the strategy proposed in this work is based on the idea that the relay can decide whether to transmit or not comparing the instantaneous SNR of the $S-R$ channel with the instantaneous SNR of the $S-D$ channel. The destination knows the direct channel state using pilots that are transmitted before data (coherent demodulation). Once it calculates the channel coefficient, a low capacity feedback channel is used. In the same way, the relay is able to calculate the source-relay coefficient directly. Thus, the relay retransmits when the instantaneous signal-to-noise ratio of the $S-R$ channel is greater than the one of the $S-D$ channel. This strategy can be written down like

$$\hat{P}_x = \begin{cases} P_x, & \text{if } \gamma_{sr} > \gamma_{sd} \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where $\hat{P}_x$ is the transmitted power at the relay.

Hence, the destination must follow two different strategies for detecting the transmitted symbol. On one hand, when the relay has not retransmitted its signal, an ML-detector is used since the destination only has one copy of the signal. Then the detected symbol is

$$x_r = \arg\min_{x \in \mathcal{A}_x} |y_{sd} - h_{sd}x|^2. \quad (5)$$

On the other hand, when the relay has retransmitted the signal, the demodulated symbol is obtained with a MRC, which can collect diversity of order two in this case. Thus the detected symbol takes the form

$$x_r = \arg\min_{x \in \mathcal{A}_x} |h_{sd}^* y_{sd} + h_{rd}^* y_{rd} - (|h_{sd}|^2 + |h_{rd}|^2)x|^2. \quad (6)$$

IV. Performance Analysis

A. Energy Saving

Since the relay only forwards the received symbol when it is required, it can save energy depending on the instantaneous SNR of the $S-D$ and $S-R$ channels. The expectation of this saving can be calculated as the probability that γ_{sd} is greater than γ_{sr}. If we call S_e the saved energy and assuming Rayleigh channels,

$$\begin{aligned} S_e(\bar{\gamma}_{sd}, \bar{\gamma}_{sr}) &= \int_0^{\infty}\int_0^{\gamma_{sd}} p(\gamma_{sr})p(\gamma_{sd})d\gamma_{sr}d\gamma_{sd} \\ &= \frac{\bar{\gamma}_{sr}}{\bar{\gamma}_{sr} + \bar{\gamma}_{sd}} \end{aligned} \quad (7)$$

Hence, DLA saves more energy as the relay gets closer to the source. When the mean values of both channels are equal, the relay saves half of its power.

B. BEP Performance for Single Relay System

For clarity in exposition, we first analyze the performance of the presented strategy for BPSK modulation. Extension to any other constellation is discussed in Section IV-C.

The received signal at the destination is driven into the MRC detector or the ML-coherent detector. With the first one, the output of the combiner is

$$y_D = \begin{cases} (|h_{sd}|^2 + |h_{rd}|^2)x + h_{sd}^* z_{sd} + h_{rd}^* z_{rd}, & \text{if } \hat{x} = x \\ (|h_{sd}|^2 - |h_{rd}|^2)x + h_{sd}^* z_{sd} + h_{rd}^* z_{rd}, & \text{if } \hat{x} = -x \end{cases} \quad (8)$$

while with the second one, the signal used for estimating the symbol is the received one in (2).

With the receivers described in (5) and (6) and defining $\underline{\gamma} := (\gamma_{sr}, \gamma_{rd}, \gamma_{sd})$, the instantaneous (BEP) is

$$P^b(\underline{\gamma}) = \begin{cases} [1 - P^b_{sr}]P^b_{rd_1} + P^b_{sr}P^b_{rd_2} & \text{if } \gamma_{sr} > \gamma_{sd} \\ P^b_{sd} & \text{if } \gamma_{sr} < \gamma_{sd} \end{cases} \quad (9)$$

where P^b_{sr} and P^b_{sd} are the instantaneous BEP of the corresponding links, and $P^b_{rd_1}$ and $P^b_{rd_2}$ are the BEP of the $R-D$ channel when $\hat{x} = x$ and $\hat{x} = -x$, respectively.

Since we are making the performance analysis for BPSK, it suffices to consider the real part of y_D, which is a gaussian random variable with variance $\sigma^2 = (|h_{sd}|^2 + |h_{rd}|^2)N_0/2$. With this consideration, bit error probabilities in (9) are

$$\begin{aligned} P^b_{rd_1} &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{[y + (|h_{sd}|^2 + |h_{rd}|^2)\sqrt{P_x}]^2}{2\sigma^2}\right\} dy, \\ P^b_{rd_2} &= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{[y + (|h_{sd}|^2 - |h_{rd}|^2)\sqrt{P_x}]^2}{2\sigma^2}\right\} dy, \\ P^b_{sd} &= \int_0^\infty \frac{1}{\sqrt{\pi\sigma^2_{sd}}} \exp - \frac{[y + |h_{sd}|\sqrt{P_x}]^2}{\sigma^2_{sd}} dy, \\ P^b_{sr} &= \int_0^\infty \frac{1}{\sqrt{\pi\sigma^2_{sr}}} \exp - \frac{[y + |h_{sr}|\sqrt{P_x}]^2}{\sigma^2_{sr}} dy. \end{aligned} \quad (10)$$

We now analyze the expected error performance of DLA. To accomplish this, it is necessary to compute the mean value of the BEP over the instantaneous signal-to-noise ratios. Using the Q-function, $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2)dt$, to express (9)-(10) in a compact way, and defining $\underline{\bar{\gamma}} := (\bar{\gamma}_{sr}, \bar{\gamma}_{rd}, \bar{\gamma}_{sd}) = (\sigma^2_{sr}\bar{\gamma}, \sigma^2_{rd}\bar{\gamma}, \sigma^2_{sd}\bar{\gamma})$, the expectation of the BEP is,

$$\begin{aligned} &P^b(\underline{\bar{\gamma}}) = \\ &\int_0^\infty \int_0^\infty \int_{\gamma_{sd}}^\infty [P_1 + P_2] p(\gamma_{sd}) p(\gamma_{sr}) p(\gamma_{rd}) \; d\gamma_{sr} d\gamma_{sd} d\gamma_{rd} + \\ &+ \int_0^\infty \int_{\gamma_{sr}}^\infty P_3 p(\gamma_{sd}) p(\gamma_{sr}) \; d\gamma_{sd} d\gamma_{sr}. \end{aligned} \quad (11)$$

where $p(\gamma_{ij})$ is the exponential probability density function (pdf) of γ_{ij}, and P_1, P_2 and P_3 are,

$$P_1 = \left(1 - Q(\sqrt{2\gamma_{sr}})\right) Q\left(\frac{\sqrt{2}(\gamma_{sd} + \gamma_{rd})}{\sqrt{\gamma_{sd} + \gamma_{rd}}}\right), \quad (12)$$

$$P_2 = Q(\sqrt{2\gamma_{sr}}) Q\left(\frac{\sqrt{2}(\gamma_{sd} - \gamma_{rd})}{\sqrt{\gamma_{sd} + \gamma_{rd}}}\right), \quad (13)$$

$$P_3 = Q(\sqrt{2\gamma_{sd}}). \quad (14)$$

To measure the goodness of our strategy, it is useful to consider the diversity gain that can be obtained with the inclusion of the relay. Diversity gain is defined as the negative exponent of the average BEP when the average SNR tends to infinity, that is, in

$$P^b(\underline{\bar{\gamma}}) \overset{\bar{\gamma}\to\infty}{\approx} (G_c\bar{\gamma})^{-G_d} \quad (15)$$

G_d is the diversity gain and G_c is the coding gain.

The next proposition for $P^b(\underline{\bar{\gamma}})$ is shown in the Appendix to hold for any $\underline{\bar{\gamma}}$.

Proposition 1 *Let $E\{P^b(\underline{\gamma})\} := P^b(\underline{\bar{\gamma}})$ be the expected BEP and $\bar{\gamma} = P_x/N_0$ the mean SNR of the system, then*

$$\lim_{\bar{\gamma}\to\infty} \frac{\log P^b(\bar{\gamma})}{\log \bar{\gamma}} = -2. \quad (16)$$

Thus the strategy presented here is full diversity achieving for a one relay system using BPSK modulation. With reference to Eq. (11), please note that, in the second term, γ_{sd} is subject to be greater than γ_{sr}, such that small values of γ_{sd} are less probable. This way, the pdf of γ_{sd} is modified, and the last term in Eq. (11) provide full diversity order with a ML-coherent detector. Similarly, in the second term of Eq. (11), the pdf of γ_{sr} is also modified by the threshold condition. This conditioning of the statistical distribution of each instantaneous SNR is crucial to prove that full diversity is achieved.

C. Extension to M-ary Constellation

Extension for higher order constellation of the performance analysis in Section IV-B is described here. We refer to [3] where a very similar analysis can be found. As in the previous section, the symbol error probability (SEP) can be expressed as the superposition of three terms, now denoted P^s_1, P^s_2 and P^s_3. P^s_1 corresponds to the case when the relay retransmits the correct symbol, whose performance analysis is the same as a co-located antenna system, in which the BEP is known to decay with an exponent of two. For the other two terms P^s_2 and P^s_3 can be bounded using the worst case in which the decoded symbol is at the maximum possible distance from the actual symbol sent by the source. In such a case, integration of the associated pdfs can be simplified by reducing the decision region to a square inscribed within the circle of radius half of the minimum distance between constellation points. Under this consideration, P^s_2 and P^s_3 are scaled versions of P_2 and P_3 respectively, with a scale factor $\log_2 M$, with M the constellation size.

D. Extension to Multi Branch System

The generalization of this strategy for a higher number of relays is discussed here. Considering Fig. 2 to describe this scenario. Please note that if for all the $S-R$ channels, we compare their instantaneous SNR only with the direct channel, only diversity of order two could be achieved, regardless of the number of relays used.

Since the key idea of this strategy is to make the relays forward their signals only when these are expected to be useful, the relay must take into account all of the other branches of Fig. 2. To carry this out, we propose the use of the approximation of the equivalent instantaneous SNR (γ_{eq_m}) described in [8], i.e. $\gamma_{eq_m} = \min\{\gamma_{sr_m}, \gamma_{rd_m}\}$, where γ_{sr_m} and γ_{rd_m} are respectively the instantaneous SNR of the $S-R$ and $R-D$ channels of the branch m.

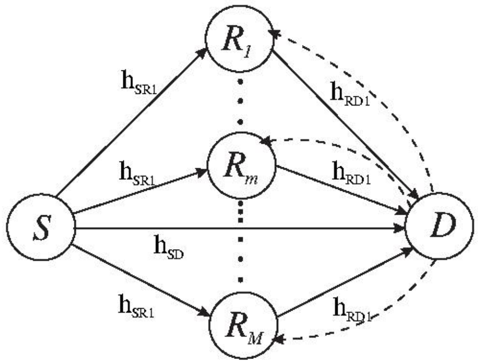

Fig. 2. Model for Multi-Branch system.

With this approximation, the threshold condition for any relay to forward or not the information is,

$$\hat{P}_{x_m} = \begin{cases} P_x, & \text{if } \gamma_{sr} \geq \max\left\{ \max\limits_{m \in \{1:M\}} \{\gamma_{eq_m}\}, \gamma_{sd} \right\} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where $\hat{P}_{x_m}$ is the transmitted power at relay m.

This generalization implies not only that full diversity of order of the number of relays plus one is achieved, but also that energy saving in each relay increases with the number of them, since as more relays are present, more difficult is to have an instantaneous SNR higher than the others.

V. Simulations and Numerical Results

Simulations to verify the claims of this paper have been run. In the first one, a performance analysis is done for some general scenarios which correspond to three different positions for the relay: 1) at the same distance from S and D, 2) close to S and 3) close to D. These scenarios are represented by the corresponding average SNRs as $(\bar{\gamma}, \bar{\gamma}, \bar{\gamma})$, $(\bar{\gamma}+30dB, \bar{\gamma}, \bar{\gamma})$ and $(\bar{\gamma}, \bar{\gamma}+30dB, \bar{\gamma})$, respectively. The BEP for this scenarios using DLA is shown in Fig. 3, in which the output BEPs are compared with the case in which no cooperation is used.

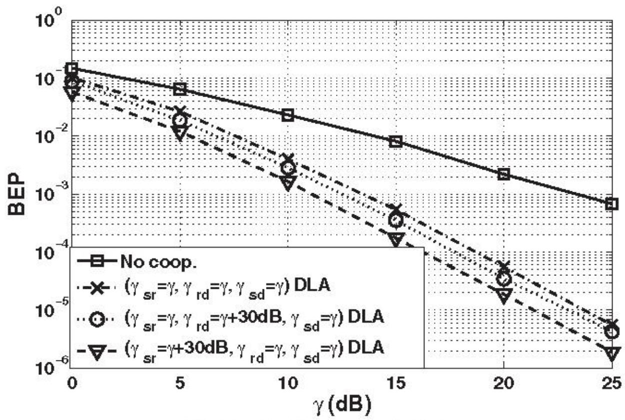

Fig. 3. Performance of DLA.

In order to compare DLA with LAR protocol, two simulations have been done. In the first one, a comparison between DLA and LAR strategies for $(\bar{\gamma}, \bar{\gamma}, \bar{\gamma})$ is presented in Fig. 4. It shows that for the same average power consumption, DLA performance is slightly better (one dB). In the second one, energy saving in the relay is measured for both protocols for five different scenarios, which correspond with five points of a curve traced by the relay from the source to the origin. Fig. 5 shows this result. The x axis represents the percentage of the full path already covered by the relay from the source. As it can be seen, when the relay is close to the destination, LAR saves more power than DLA, but in the rest of the cases DLA performs better in energy-saving sense.

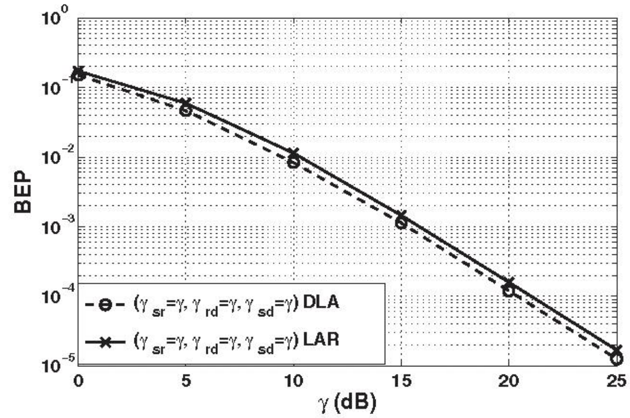

Fig. 4. Performance comparison using DLA vs. LAR.

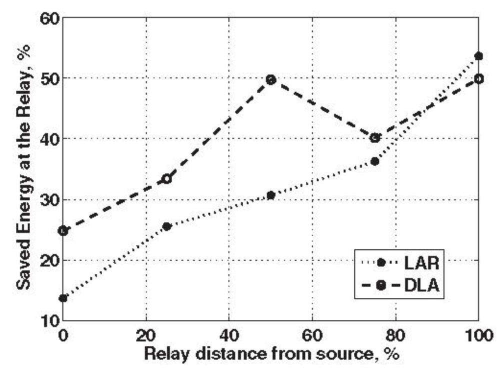

Fig. 5. Saved Energy for DLA and LAR.

Finally, a simulation is driven for showing the performance of the Multi-Branch case, with one, two and three relays for the equidistance scenario. The result is shown in Fig. 6. It can be observed that full diversity with order up to the number of relays plus one is achieved.

VI. Conclusions

An on-off strategy for cooperative communications is presented in which signal is forwarded by the relay according to the state of the source-destination channel, in such a way that the relay only consumes power when its support is really needed and when its retransmission is expected to be reliable.

Extension to the Multi-Branch case is possible using an approximation of the equivalent SNR of each branch. Analytical

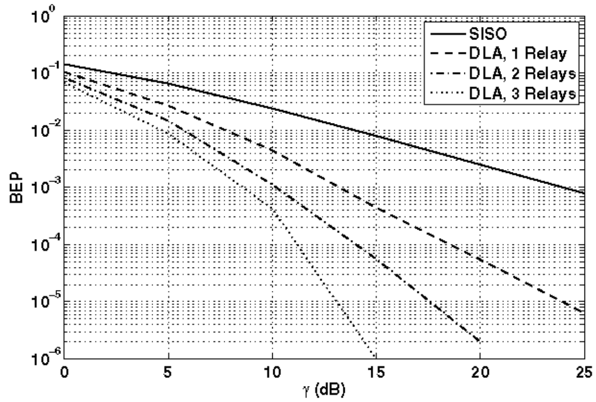

Fig. 6. Performance of Multi-Branch generalization.

performance claims show that full diversity and energy savings are obtained.

Simulations have been run, where the diversity gain is observed, and where the protocol is compared to another cooperative strategy (LAR). For the same BEP, in the equidistance case, DLA saves slighly more energy (1 dB) than LAR. Comparing the saved energy for different scenarios, DLA shows to save more energy in almost any set-ups.

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Appendix

To prove Proposition 1 it is enough to show that both terms in (11) decay with an exponent that at least equals two for high SNR. Let us start by the second summand,

$$A \leq \int_0^{\infty}\int_{\gamma_{sr}}^{\infty} P_3 p(\gamma_{sd})p(\gamma_{sr})\ d\gamma_{sd}d\gamma_{sr}. \qquad (18)$$

Using Chernoff bound it is straightforward that,

$$\begin{aligned} A &= \int_0^{\infty}\int_{\gamma_{sr}}^{\infty} \frac{1}{2\bar{\gamma}_{sd}\bar{\gamma}sr} \exp\left(-\gamma_{sd}\left(1+\frac{1}{\bar{\gamma}_{sd}}\right)\right) \\ & \quad \exp\left(-\frac{\gamma_{sr}}{\bar{\gamma}_{sr}}\right)\ d\gamma_{sd}d\gamma_{sr}. \end{aligned} \qquad (19)$$

After integrating over different regions, we arrive at,

$$A \leq \frac{1}{2}\frac{1}{\bar{\gamma}^2\sigma_{sd}^2\sigma_{sr}^2 + \bar{\gamma}(\sigma_{sd}^2+2\sigma_{sr}^2)+1+\frac{1}{\bar{\gamma}\sigma_{sr}^2}} \overset{\bar{\gamma}\to\infty}{=} (k_1\bar{\gamma})^{-2} \qquad (20)$$

where k_1 is a constant which depends on σ_{sd}^2 and σ_{sr}^2.

Now let us consider the first summand in (11), which can be expanded as the sum of the terms $B+C$, which take the form

$$\begin{aligned} B &= \int_0^{\infty}\int_0^{\infty}\int_{\gamma_{sd}}^{\infty}\Big(1-Q\big(\sqrt{2\gamma_{sr}}\big)\Big)\, Q\left(\frac{\sqrt{2}(\gamma_{sd}+\gamma rd)}{\sqrt{\gamma_{sd}+\gamma_{rd}}}\right)\times \\ & \quad \times p(\gamma_{sd})p(\gamma_{sr})p(\gamma_{rd})\ d\gamma_{sr}d\gamma_{sd}d\gamma_{rd} \end{aligned} \qquad (21)$$

$$\begin{aligned} C &= \int_0^{\infty}\int_0^{\infty}\int_{\gamma_{sd}}^{\infty}\Big(Q\big(\sqrt{2\gamma_{sr}}\big)\Big)\, Q\left(\frac{\sqrt{2}(\gamma_{sd}-\gamma_{rd})}{\sqrt{\gamma_{sd}+\gamma_{rd}}}\right)\times \\ & \quad \times p(\gamma_{sd})p(\gamma_{sr})p(\gamma_{rd})\ d\gamma_{sr}d\gamma_{sd}d\gamma_{rd} \end{aligned} \qquad (22)$$

For B, likewise for A, using Chernoff bound and integrating, it is easy to arrive at,

$$B \leq \frac{1}{2}\frac{k_2}{k_3\bar{\gamma}^2+k_4\bar{\gamma}+k_5} \overset{\bar{\gamma}\to\infty}{=} (k_6\bar{\gamma})^{-2} \qquad (23)$$

where k_2,k_3,k_4,k_5,k_6 are constants which depend on σ_{sd}^2, σ_{sr}^2 and σ_{rd}^2.

Finally, to prove that C decays with exponent two we use the Chernoff bound applied to the first term in γ_{sr} and integrate over this variable,

$$\begin{aligned} C &\leq \frac{1}{(\bar{\gamma}_{sr}+1)\bar{\gamma}_{sd}}\int_0^{\infty}\int_0^{\infty} Q\left(\frac{\sqrt{2}(\gamma_{sd}-\gamma_{rd})}{\sqrt{\gamma_{sd}+\gamma_{rd}}}\right)\times \\ & \times \exp\left(-\gamma_{sd}\left(1+\frac{1}{\bar{\gamma}_{sr}}\right)\right) p(\gamma_{sr})p(\gamma_{rd})\ d\gamma_{sd}d\gamma_{rd} \end{aligned} \qquad (24)$$

As the Q function is always minor than one, (24) is bounded by

$$\begin{aligned} C &\leq \frac{1}{(\bar{\gamma}_{sr}+1)\bar{\gamma}_{sd}}\int_0^{\infty}\int_0^{\infty}\exp\left(-\gamma_{sd}\left(1+\frac{1}{\bar{\gamma}_{sr}}\right)\right) \\ & p(\gamma_{sr})p(\gamma_{rd})\ d\gamma_{sd}d\gamma_{rd} = \\ &= \frac{1}{(\bar{\gamma}_{sr}+1)\bar{\gamma}_{sd}}\int_0^{\infty}\exp\left(-\gamma_{sd}\left(1+\frac{\bar{\gamma}_{sr}+\bar{\gamma}_{rd}}{\bar{\gamma}_{sr}\bar{\gamma}_{rd}}\right)\right)d\gamma_{sd} \\ &= \frac{\bar{\gamma}_{sd}\bar{\gamma}_{sr}}{\bar{\gamma}_{sd}(\bar{\gamma}_{sr}+1)(\bar{\gamma}_{sd}\bar{\gamma}_{sr}+\bar{\gamma}_{sd}+\bar{\gamma}_{sr})} \overset{\bar{\gamma}\to\infty}{=} (k_7\bar{\gamma})^{-2} \end{aligned} \qquad (25)$$

where k_7 is a constant which depends on σ_{sd}^2, σ_{sr}^2 and σ_{rd}^2.