A New Method for Single-Step Robust Post-Processing of Flow Color Doppler M-Mode Images Using Support Vector Machines

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Abstract

Intra-cardiac pressure gradients (ICPG) are usually estimated by post-processing of flow Color Doppler M-mode images (CDMMI) by using a sequence of processing steps. We propose a novel image processing method which gives a single-step approximation of the ICPG image, based on a simple, yet specifically developed, Support Vector Machine (SVM) algorithm. Our method only requires the SVM estimation of the blood velocity from the CDMMI. Given that ICPG images are obtained by deterministic operators (Euler's momentum equation) on the blood velocity, the ICPG estimation is a simple model that consists of the same coefficients and the operator applied to the Mercer's kernel. A diverse-width Mercer's kernel is proposed, as an alternative to conventional Radial Basis Function kernel. Simulations on a synthetic model and approximations of a real example image, trained with up to 10% of the pixels, show the possibilities of this new single-step post-processing method.

1. Introduction

Doppler echocardiography is probably the most useful noninvasive technique to assess cardiovascular function [1]. Not only the blood velocity, but also intracardiac pressure differences can be obtained noninvasively by using ultrasound images under certain conditions. The simplified Bernouilli equation yields the transvalvular pressure difference from Doppler measurements of blood jet velocity. More recently, the noninvasive estimation of intracardiac pressure gradients (ICPG) from color Doppler M-mode images (CDMMI) has been successfully addressed, which allows the estimation of a number of clinically useful cardiac indices [2, 3, 4]. Mathematical derivation of color-Doppler velocity-data is strongly dependent on noise and artifacts, and indices numerically derived from these images are heavily influenced by the lack of signal ("black holes") and by the noise. Conventional approaches to ICPG estimation use a two-steps procedure: first, CDMMI is restored (splines or image filtering), and second, the necessary operators are applied to provide with ICPG image (spline-based transformations or numerical image processing).

We propose to make a single-step, robust approximation to ICPG images by using Support Vector Machines (SVM) [5]. As it will be seen, the final expression for the CDMMI approximation with SVM has an easy to handle expression, as it depends only on a sparse set of coefficients and the used Mercer's kernel. Therefore, ICPG estimation can be readily obtained from the same set of coefficients as the CDMMI estimation and by simple operators transforming the kernel.

The draw of the paper is as follows. In the next section, the analytical equations for the noninvasive derivation of ICPG images from CDMMI are introduced. Then, the SVM model is proposed and developed. Simulations and application on an image example are presented.

2. ICPG estimation from CDMMI

The estimation of ICPG from flow CDMMI is based on the one-dimensional Euler's momentum equation. This expression represents the balance between the driving pressure force p and the inertial and convective forces associated with acceleration of a fluid along a linear streamline s

[2], this is:

$$\frac{\partial p(s,t)}{\partial s} = -\rho\left(\frac{\partial v_b(s,t)}{\partial t} + v_b(s,t)\frac{\partial v_b(s,t)}{\partial s}\right) \quad (1)$$

where $v_b(s,t)$ is the blood velocity along the streamline, assuming one-dimensional flow propagation, and ρ is the blood flow density (usually $\rho = 1.05$). If Doppler interrogation is fully coaxial to flow, a color Doppler M-mode recording provides the full spatio-temporal velocity distribution of the streamline. With this method, the ICPG can be calculated in the absence of a restrictive orifice, thus providing us with the real driving forces of flow within the heart. Two terms can be distinguished in (1), which are

$$\delta p^i(s,t) = -\rho\frac{\partial v_b(s,t)}{\partial t} \quad (2)$$

$$\delta p^c(s,t) = -\rho v_b(s,t)\frac{\partial v_b(s,t)}{\partial s} \quad (3)$$

known as inertial and convective pressure gradients, respectively, so that $\frac{\partial p(s,t)}{\partial s} = \delta p^i(s,t) + \delta p^c(s,t)$.

By spatial integration of (1), instantaneous pressure difference between any two intracardiac stations along the streamline can be obtained. For instance, if we choose the left atrium and the LV apex, and by constraining the observation time to diastole, we obtain the full LV transmitral filling ICPG [3]. In this paper, we will limit to the estimation of ICPG images, the obtention of pressure difference curves being straightforward.

3. SVM model for ICPG image estimation

The proposed image model for ICPG estimation uses a dual signal model [5] for flow CDMMI approximation. Let $v_b(s,t)$ and $\{V_{i,j} = v(i\delta s, j\delta t); i = 1,\cdots,N_s; j = 1,\cdots,N_t\}$ denote the velocity field and the acquired image ($N_t \times N_s$ matrix), respectively. Also, let $[i,j]$ denote the image coordinates of pixel $V_{i,j}$, and let I denote the set of coordinates for all the image pixels. Then, by using some criterion, I can be split into subsets, I^{train} and I^{test}, to be used for training and testing the model.

The SVM model for CDMMI estimation uses the following expression for nonlinear regression of each pixel as a function of a nonlinear transformation of its image coordinates:

$$V_{i,j} = < \boldsymbol{w}, \boldsymbol{\phi}([i,j]) > + b + e_{i,j} \quad (4)$$

with $[i,j] \in I^{train}$, where $e_{i,j}$ is the model approximation error for the pixel; $\boldsymbol{\phi}([i,j])$ is a nonlinear application of coordenite vector $[i,j]$ to a high-dimensional (say P-dimensional) feature space $\mathfrak{F}$; and b is a bias term. A linear regression for the pixel value is given by the dot product of nonlinearly transformed pixel coordinates and $\boldsymbol{w} \in \mathfrak{F}$.

Given this image model, we propose to use the ε-Huber robust cost [6], which is given by

$$L(e_{i,j}) = \begin{cases} 0, & |e_{i,j}| \leq \varepsilon \\ \frac{1}{2\delta}(|e_{i,j}| - \varepsilon)^2, & \varepsilon \leq |e_{i,j}| \leq e_C \\ C(|e_{i,j}| - \varepsilon) - \frac{1}{2}\delta C^2, & |e_{i,j}| \geq e_C \end{cases} \quad (5)$$

where $e_C = \varepsilon + \delta C$; ε is the insensitive parameter, and δ and C control the trade-off between regularization and losses. The ε-insensitive zone ignores errors lower than ε; quadratic cost zone uses the L_2-norm of errors, which is appropriate for Gaussian noise; and linear cost zone limits the effect of outliers. By following the conventional SVM methodology, the previous loss function is regularized with the squared norm of model coefficients, and primal problem consists of minimizing

$$\frac{1}{2}\sum_{p=1}^{P} w_p^2 + \frac{1}{2\delta}\sum_{[i,j]\in I_1^{train}} (\xi_{i,j}^2 + \xi_{i,j}^{\star 2}) + + C\sum_{[i,j]\in I_2^{train}} (\xi_{i,j} + \xi_{i,j}^{\star}) - \sum_{[i,j]\in I_2^{train}} \frac{\delta C^2}{2} \quad (6)$$

with respect to w^p, $\{\xi_{i,j}^{(\star)}\}$ (notation for both $\{\xi_{i,j}\}$ and $\{\xi_{i,j}^{\star}\}$), and b, and constrained to

$$V_{i,j} - < \boldsymbol{w}, \phi([i,j]) > -b \leq \varepsilon + \xi_{i,j} \quad (7)$$

$$-V_{i,j} + < \boldsymbol{w}, \phi([i,j]) > +b \leq \varepsilon + \xi_{i,j}^{\star} \quad (8)$$

and to $\xi_{i,j}, \xi_{i,j}^{\star} \geq 0$, for $[i,j] \in I^{train}$; $\{\xi_{i,j}^{(\star)}\}$ are *slack variables* or *losses*, and they handle the residuals according to the robust cost function; and I_1^{train}, I_2^{train} are the subsets of pixels for which losses are in the quadratic or in the linear cost zone, respectively.

Similar derivations of the dual functional can be found in the literature [5, 6]. In brief, by including constraints (7),(8) into (6), the primal-dual functional (or Lagrange functional) is obtained. By making zero the gradient of the Lagrangian with respect to the primal variables [6], and by using the Karush-Khun-Tucker conditions, several manipulations can be done. The correlation matrix of input space pixel pairs can be identified, and denoted as $\boldsymbol{R}([i,j],[k,l]) \equiv < \boldsymbol{\phi}([i,j]), \boldsymbol{\phi}([k,l]) >$. The dual problem can now be obtained and expressed in matrix form, as the maximization of

$$-\frac{1}{2}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^{\star})^T [\boldsymbol{R} + \delta\mathbf{I}] (\boldsymbol{\alpha} - \boldsymbol{\alpha}^{\star}) + (\boldsymbol{\alpha} - \boldsymbol{\alpha}^{\star})^T \boldsymbol{V} - - \varepsilon\mathbf{1}^T(\boldsymbol{\alpha} + \boldsymbol{\alpha}^{\star}) \quad (9)$$

constrained to $C \geq \alpha_{i,j}^{(\star)} \geq 0$, where $\alpha_{i,j}, \alpha_{i,j}^{*}$ are the Lagrange multipliers corresponding to (7),(8); and $\boldsymbol{\alpha}^{(\star)} = [\alpha_{i,j}^{(\star)}]$, $\boldsymbol{V} = [V_{i,j}]$, for $[i,j] \in I^{train}$ are column vectors.

After obtaining $\boldsymbol{\alpha}^{(\star)}$, the velocity for a pixel at $[k,l]$ is

$$\hat{V}_{k,l} = \sum_{[i,j]\in I^{train}} \beta_{i,j} < \boldsymbol{\phi}([i,j]), \boldsymbol{\phi}([k,l]) > +b \quad (10)$$

with $\beta_{i,j} = \alpha_{i,j} - \alpha^{\star}_{i,j}$, which is a weighted function of the nonlinearly observed times in the feature space. Note that only a reduced subset of the Lagrange multipliers is nonzero, which are called the *support vectors*, and the CDMMI estimation is built only with them.

A Mercer's kernel is a bivariate function that is equivalent to the calculation of a dot product in $\mathfrak{F}$ [5], this is, $K([i,j],[k,l]) =< \boldsymbol{\phi}([i,j]), \boldsymbol{\phi}([k,l]) >$. However, we do not need to know explicitly neither $\mathfrak{F}$ nor the nonlinear application, but still the dot products in $\mathfrak{F}$ can be readily calculated with the kernel. A usual nonlinear Mercer's kernel is the *Gaussian (RBF) kernel*, given by $K_G([i,j],[k,l]) = \exp\left(\frac{\|[i,j]-[k,l]\|^2}{-2\sigma^2}\right)$ where σ is the width parameter. However, given that different magnitudes (space and time) are involved, we propose the following (D-RBF) kernel for M-mode Doppler images:

$$K_D([i,j],[k,l]) = \exp\left(\frac{|i-k|^2}{-2\sigma_s^2}\right)\exp\left(\frac{|j-l|^2}{-2\sigma_t^2}\right)$$

where σ_s, σ_t account for the scaling of spatial and temporal dimensions, respectively.

Thus, the CDMMI model can finally be expressed as

$$\hat{V}_{k,l} = \sum_{[i,j]\in I^{train}} \beta_{i,j} K([i,j],[k,l]) + b \quad (11)$$

and the expression for the estimated inertial pressure gradient will be simply given by

$$\hat{\delta}^{i}_{k,l} = -\rho \sum_{[i,j]\in I^{train}} \beta_{i,j} K^t([i,j],[k,l]) \quad (12)$$

whereas the estimated convective component will be

$$\hat{\delta}^{c}_{k,l} = -\rho \left(\sum_{[i,j]\in I^{train}} \beta_{i,j} K([i,j],[k,l]) + b \right) \cdot \cdot \left(\sum_{[m,n]\in I^{train}} \beta_{m,n} K^s([m,n],[k,l]) \right) \quad (13)$$

where K^t, K^s denote the partial derivatives of the kernel, with respect to time and space, respectively.

Finally, recall that several free parameters need to be adjusted, namely, the width(s) of Gaussian kernel(s), and the free parameters of the cost function (ε, δ, C). In our case, an intra-image strategy is used, that consists on using the pixels corresponding to I^{test} to determine the best free parameters for the image at hand.

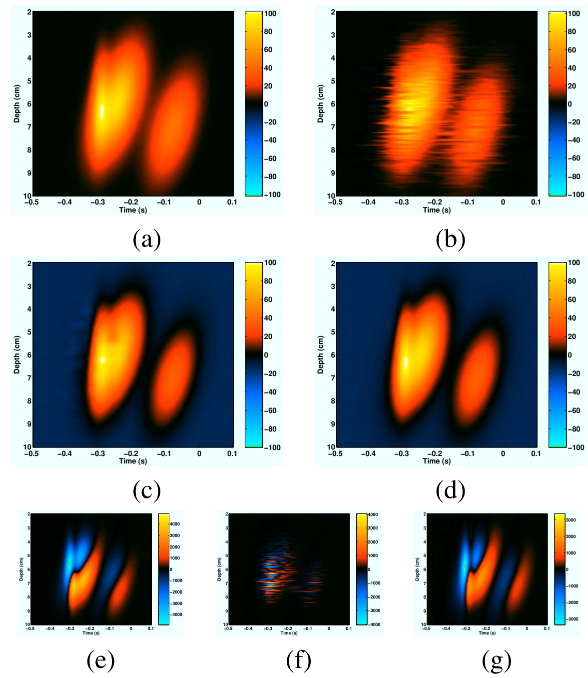


Figure 1. Symbolic CDMMI (a), and approximation (5% training pixels) with splines (b), RBF kernel (c) and D-RBF kernel (d). Symbolic (normalized) ICPG image (e), and estimation with splines (f) and D-RBF kernel (g).

4. Experiments and results

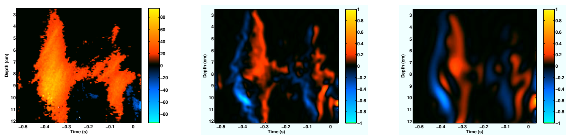
Image model. A simple model of diastolic transmitral flow CDMMI was created as

$$v_b(s,t) = \sum_{r=1}^{3} a_i \exp\left\{ -\frac{1}{2}[s_r,t_r]\Sigma_r^{-1}[s_r,t_r]^T \right\} \quad (14)$$

where $[s_r, t_r]$ is a bidimensional row vector, Σ_r is the covariance matrix of each component, and s_r, t_r, Σ_r, are given in Table 1. As shown in Fig. 1(a), two components account for early LV filling (E-wave; $i = 1,2$), and a lower amplitude component emulates late filling (A-wave; $i = 3$). Parameters were adjusted to match physiological values of both waves, and time 0 was defined at the QRS onset. By using symbolic calculations, Euler's equation was solved for the Gaussian mixture velocity field to pro-

r	1	2	3
t_r	$\frac{t+0.25}{0.05}$	$\frac{t+0.08}{0.05}$	$\frac{t+0.25}{0.05}$
a_r	90	45	48.75
s_r	$\frac{s-6}{1.5}$	$\frac{s-7}{1.5}$	$\frac{s-6}{1.5}$
Σ_i	$\begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.8 & -0.5 \\ -0.5 & 0.1 \end{pmatrix}$

Table 1. *Constant parameter values of color-Doppler transmitral flow model (a_r in cm/s, t_r in s).*



(a) (b) (c)

Figure 2. CDMMI in a volunteer (a), (normalized) ICPG image with splines (all samples) (b), and D-RBF SVM with 5% of training samples (c).

vide with the theoretical values of ICPG images.

Results with image model. Digital images were simulated from the blood flow velocity model, using finite resolutions of 200 Hz and 20 pixels/cm. This gave the gold-standard for both CDMMI and ICPG image (121×161 pixels). A spline-based algorithm was used to build first the CDMMI, and then to calculate the ICPG by cubic spline-based operators. Also, SVM algorithm was used with both RBF and D-RBF kernels. The same training subset of samples (random image subsampling) was used to build the CDMMI in all the methods, and mean square error (MSE) of approximations was calculated in the test pixels.

In Fig. 1, symbolic images and approximations are shown. Table 2 shows the MSE in CDMMI as a function of the training pixels. It can be observed that for increasing ratio of training pixels, the overall velocity reconstruction improves at a higher rate in SVM, specially for D-RBF kernel.

Example of CDMMI. The CDMMI from a healthy volunteer is shown in Fig. 2(a). Scanner resolutions for acquisition were 600 Hz, 26.2 pixels/cm, and 5 bits (equivalent to 1.25 cm/s of blood velocity resolution). The 126×171 image was subsampled by 2, and 5% of training pixels were used. MSE for CDMMI was 86.25 using splines and 34.55 using D-RBF SVM. Figure 2(b,c) show the corresponding ICPG image estimations. These results are qualitatively coherent with the trends shown by synthetic image experiments. For a fair comparison, D-RBF SVM has to be trained with the full image, rather than with a subset of pixels, given that currently used spline techniques use the full image. For this purpose, large-scale SVM algorithms need to be used and adapted to CDMMI approximation.

$\sharp\ tr.pixels$	1.2%	2.5%	5%	10%
Spline	147.2	96.3	46.2	20.6
RBF	14.9	3.06	0.87	0.064
D-RBF	0.46	0.012	5.7e-3	5.3e-4

Table 2. MSE in CDMMI approximation.

5. Conclusions

A new method for single-step estimation of ICPG images from CDMMI has been presented. The method uses nonlinear SVM with D-RBF kernel, which is appropriate for M-mode Doppler images. Comparisons with spline-based processing with reduced number of training pixels (10%) show the performance of SVM under these conditions. The immediate future work is devoted to the development of schemes working with available large-scale and fast SVM algorithms [5], for a fair comparison with currently used spline techniques. Also, the inclusion of physiological constrains and the application to other ultrasound image modalities will be explored.

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