Power-Efficient OFDM via Quantized Channel State Information*

Antonio G. Marques†, *Fadel F. Digham, and Georgios B. Giannakis*

Department of ECE
University of Minnesota
Minneapolis, MN 55455, USA

***Abstract*— In response to the growing demand for low-cost low-power wireless sensor networks and related applications, we develop bit and power loading algorithms that minimize transmit-power for orthogonal frequency division multiplexing (OFDM) under rate and error probability constraints. Our novel algorithms exploit one of three types of channel state information at the transmitter (CSIT): deterministic (per channel realization) for slow fading links, statistical (channel mean) for fast fading links, and quantized (Q-) CSIT whereby a limited number of bits are fed back from the transmitter to the receiver. By adopting average transmit-power as a distortion metric, a channel quantizer is also designed to obtain a suitable form of Q-CSI. Numerical examples corroborate the analytical claims and reveal that significant power savings result even with a few bits of Q-CSIT.**

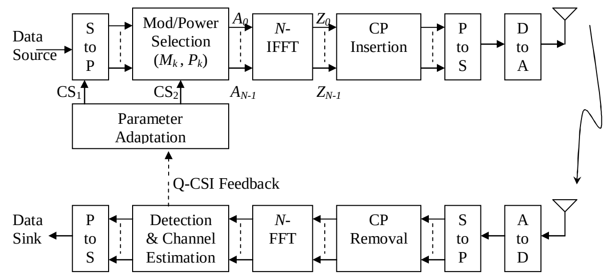

Fig. 1. Transmission System Block diagram.

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been known for its spectral efficiency, error resilience, and ease in implementation. For these merits, OFDM has been adopted in many applications such as digital subscriber line (DSL), digital audio and video broadcasting (DAB/DVB) standards and wireless local area networks, to name a few [7], [13]. Unlike maximum-rate or minimum bit error rate (BER) driven applications (see e.g., [4], [8] and references therein), this paper is tailored for minimum power applications that have been recently receiving much attention in both commercial and tactical communication systems. Towards this objective, we rely on channel state information at transmitter (CSIT), to design optimum adaptive power and bit loading schemes that minimize transmit-power for OFDM under prescribed data rate and BER. This is in contrast to spectrally-efficient OFDM designs which maximize rate for a prescribed transmit power.

Three types of CSIT will be considered. The first is deterministic CSIT (D-CSIT) that has been used in DSL and can also be available the wireless channel is fading slowly. However, D-CSIT may be unrealistic to assume available in many wireless scenarios due to channel estimation errors at the receiver, feedback delay and asymmetry between forward and reverse links. This has motivated OFDM loading schemes based on statistical (S) or quantized (Q) CSIT, whereby a limited number of bits are fed back from the transmitter to the receiver; see e.g., [8], [14] and references therein.

*The work in this paper was supported by the USDoD ARO grant No. W911NF-05-1-0283.The work of the first author was supported by the Spanish Government TEC2005-06766-C03-01/TCM.

†The first author is with TSC, Rey Juan Carlos Univ., 28943 Fuenlabrada, Madrid, Spain.

Interestingly, except for [11] where D-CSIT is used to minimize power consumption, OFDM has not been optimized in the *power-limited* regime. It is this paper's main goal to design power-efficient OFDM transceivers for frequency selective channels using Q-CSIT. We further consider the two limiting scenarios of either slow or fast fading for which we design power efficient OFDM schemes based on D-CSIT or S-CSIT. Our Q-CSIT based scheme will turn out to be much more power-efficient than the S-CSIT based one, and for a sufficient number of feedback bits it approaches the power savings achieved by the benchmark D-CSIT design.

The rest of the paper is organized as follows. Section II describes the setup. Power-efficient OFDM designs using D-CSIT, Q-CSIT, or S-CSIT are studied in Sections III, IV, and V, respectively. Finally, numerical results are provided in Section VI.

II. System Overview

We consider a point-to-point wireless OFDM transmission using N sub-carriers (sub-channels) through a frequency-selective channel with discrete-time baseband equivalent impulse response taps $\{h_n\}_{n=0}^{L}$, where: $L := \lfloor D_{\max} W \rfloor$ denotes the channel order, $D_{\max}$ its maximum delay spread, W the transmit-bandwidth, and $\lfloor \cdot \rfloor$ stands for the floor operation; see also Figure 1. A serial stream of data bits is first de-multiplexed to form N parallel streams each of which (indexed by $k \in [0, N-1]$) is multiplied by an appropriate constant to load power P_k, and modulated to obtain symbols $\{A_k\}_{k=0}^{N-1}$ drawn from a constellation of size M_k. The N-point inverse fast Fourier transform (I-FFT) applied to each snapshot of the N symbol streams is followed by insertion of a size-L cyclic prefix (CP) to yield a block of $N + L$ symbols (a.k.a. OFDM symbol), which are subsequently multiplexed and digital to analog (D/A) converted for transmission. These operations along with the corresponding FFT and CP removal at the receiver convert the multipath fading frequency-selective channel to a set of N parallel flat-fading sub-channels each with fading coefficient given by the FFT:

$$H_k = (1/\sqrt{N}) \sum_{n=0}^{N-1} h_n e^{-j\frac{2\pi}{N}kn}, \tag{1}$$

where N is typically chosen so that $N \gg L$.

Periodically inserted training symbols (a.k.a. pilots) facilitate channel estimation at the receiver side. With the channel acquired, the receiver has available a noise-normalized channel gain vector $\mathbf{g} := [g_0, \ldots, g_{N-1}]^T$, where $[.]^T$ denotes transposition and $g_k := |H_k|^2/\sigma_k^2$ is the instantaneous noise-normalized channel gain of the kth sub-channel on which the zero-mean additive white Gaussian noise (AWGN) has variance σ_k^2. While each deterministic realization of the random gain g_k constitutes the D-CSI, its statistical average $\bar{g}_k := \mathbb{E}[g_k]$ is what we henceforth refer to as S-CSI of the kth sub-channel. Either one must be available to the transmitter in the D-CSIT or S-CSIT based OFDM designs. For Q-CSIT based design, sole knowledge of $\bar{g}_k$ specifies the implemented channel quantizer whereas both $\bar{g}_k$ and quantized g_k determine bit and power allocation. The quantizer design entails selection of quantization thresholds $\tau_{k,j}$, which in turn define quantization regions $\mathcal{R}_{k,j}$ each sub-channel realization g_k falls into. The random vector of bits $\mathbf{j} := [j_0, j_1, \ldots, j_{N-1}]^T$ representing indices of the regions of all sub-channels is what we term Q-CSI.

Given either $\mathbf{g}$ (D-CSIT), or its mean $\bar{\mathbf{g}}$ (S-CSIT), or the quantized form of the channel gains specified by the bit vector $\mathbf{j}$ (Q-CSIT), our ultimate goal is to choose power and bit vectors $\mathbf{p} := [P_0 \ldots P_{N-1}]^T$ and $\mathbf{m} := [M_0 \ldots M_{N-1}]^T$ so as to minimize the power $P := \sum_{k=0}^{N-1} P_k$, or its average $\bar{P}$, for prescribed values of the rate (R_0) and the bit error rate (BER_0) per OFDM symbol $\mathbf{a} := [A_0 \ldots A_{N-1}]^T$. In addition, we wish to design a proper channel quantizer used by the receiver to provide the Q-CSI to the transmitter through the bit vector $\mathbf{j}$. We will pursue these objectives under the following assumptions:

a1. *Symbols A_k are drawn from quadrature amplitude modulation (QAM) constellations of size M_k;*

a2. *Sub-channels H_k remain invariant over at least two consecutive OFDM symbols, they are allowed to be correlated, and each is complex Gaussian distributed; i.e., each sub-channel gain g_k adheres to an exponential probability density function (pdf)*

$$f_{g_k}(g_k) = (1/\bar{g}_k)\exp(-g_k/\bar{g}_k); \tag{2}$$

a3. *Feedback is error-free and incurs negligible delay.*

Assumptions a1-a3 are common to existing designs [7], [4], [8], and are typically satisfied in practice. In principle, our results apply to any channel pdf, but a2 simplifies the resultant designs. Channel invariance in a2 allows for feedback delays; it may be stringent for D-CSIT based designs, but is very reasonable in the Q-CSIT case since $\mathbf{g}$ is allowed to vary from one OFDM symbol to the next so long as $\mathbf{j}$ remains invariant. Finally, error-free feedback is easily guaranteed with sufficiently strong error control codes especially since rate in the reverse link is low.

With R denoting rate (number of bits per OFDM symbol), the instantaneous BER is

$$\text{BER} = \frac{1}{R}\sum_{k=0}^{N-1} \log_2(M_k)\text{BER}_k(g_k, P_k, M_k) \tag{3}$$

where for the QAM constellations in a1, we have

$$\text{BER}_k(g_k, P_k, M_k) \simeq a\exp\left(-\beta_k g_k P_k\right) \tag{4}$$

and $a = 0.2$, $\beta_k := \frac{b_k}{M_k - 1}$, with $b_k = 1$ if $M_k = 2$, or, $b_k = 1.5$ if $M_k \geq 4$. The approximation of the BER_k corresponding to the kth sub-channel is very accurate [3], [5].

III. Power Optimization based on D-CSIT

In this section, capitalizing on the D-CSIT gain vector $\mathbf{g}$, we optimize power efficiency of OFDM for slow fading wireless or wireline channels given constraints on data rate and BER. In addition, the resultant performance based on the full CSI $\mathbf{g}$ serves as a benchmark for other designs that exploit partial (namely quantized or statistical) CSI. With $\mathcal{M}$ denoting the finite set of possible values of $\mathbf{m}$, our objective can be formulated as:

$$\begin{aligned} &\min_{\mathbf{m}\in\mathcal{M},\mathbf{p}(\mathbf{g})\geq\mathbf{0}} J(\mathbf{p}),\ \ J(\mathbf{p}) := \textstyle\sum_{k=0}^{N-1} P_k(g_k) \\ &\text{s. to} \\ &\text{C1.}\ \textstyle\sum_{k=0}^{N-1} \frac{\log_2(M_k)}{R_0}\text{BER}_k(g_k, P_k, M_k) = \text{BER}_0 \\ &\text{C2.}\ \textstyle\sum_{k=0}^{N-1} \log_2 M_k = R_0\,. \end{aligned} \tag{5}$$

Since entries in $\mathbf{p}$ take on continuous values while those in $\mathbf{m}$ are integer-valued, we deal with a mixed-integer programming (MIP) problem, which can be solved in two steps. First, for a fixed $\mathbf{m}$, we remove C2 and thus solve (5) to obtain an optimum $\mathbf{p}$ expressed in terms of $\mathbf{m}$. Then, we test all feasible entries of $\mathbf{m}$ that satisfy C2 to eventually select the overall optimum $(\mathbf{p}, \mathbf{m})$ pair. Substituting (4) into (5), the first step is:

$$\begin{aligned} &\min_{\mathbf{p}(\mathbf{g})\geq\mathbf{0}} J(\mathbf{p}),\ \ J(\mathbf{p}) = \textstyle\sum_{k=0}^{N-1} P_k(g_k) \\ &\text{s. to} \\ &\text{C1.}\ \textstyle\sum_{k=0}^{N-1} \frac{a\log_2(M_k)}{R_0} e^{-\beta_k g_k P_k(g_k)} = \text{BER}_0. \end{aligned} \tag{6}$$

Invoking the necessary Karush-Kuhn-Tucker (KKT) conditions for optimality [6, Sec. 14.6], the solution $P_k(g_k)$ can be found to be:

$$P_k(g_k) = \frac{1}{\beta_k g_k}\left[\ln\left(\frac{a\beta_k g_k \log_2(M_k)}{R_0}\lambda_D\right)\right]^+, \tag{7}$$

where $[x]^+ := \max(x, 0)$, and λ_D is the so-called Lagrange multiplier which can be obtained after plugging (7) into C1. Due to the convexity of both $J(\mathbf{p})$ and C1 over $\mathbf{p}$, the solution in (7) is globally optimum. Reminiscent of the water-filling principle, (7) also shows that depending on sub-channel gains, it is possible for certain sub-channels to receive zero-power. Let $\mathcal{N}_a$ be the set of indices corresponding to the $N_a \leq N$ active sub-channels which are allocated non-zero power. Upon substituting (7) into C1, we obtain $\lambda_D = \frac{1}{\text{BER}_0}\sum_{k\in\mathcal{N}_a}\frac{1}{\beta_k g_k}$.

We next incorporate the rate constraint C2 in the second step of our optimization process, and reduce the search over $\mathbf{m}$ considerably by sorting the channel gains in descending order and then assigning $M_{k_1} \geq M_{k_2}$ if $g_{k_1} \geq g_{k_2}$; see also [11]. The rationale behind this rule is that the higher g_k is, the higher M_k can be loaded to minimize P.

Remark 1: In the optimal loading schemes of this section as well as those of the ensuing sections we do not impose peak-to-average-power-ratio (PAPR) constraints. However, available digital pre-distortion schemes (such as the selected mapping algorithm in [12]) can be applied to each block of $\{A_k\}_{k=0}^{N-1}$ symbols in order to meet pre-specified PAPR constraints.

In the next section, we develop a power efficient OFDM scheme using Q-CSIT; i.e., only a few feedback bits.

IV. Optimization based on Q-CSIT

Unlike optimizing power efficiency of OFDM based on the realization $\mathbf{g}$ carried out in the preceding section, here we fulfill the same objective while relying on the bit vector $\mathbf{j}$ comprising codewords j_k which represent the indices of the quantization regions each sub-channel gain g_k falls into. In this context, we will assume that:

a4. *Codeword* j_k *has length* B $\forall k$*; hence,* $\mathbf{j}$ *is an* $NB \times 1$ *vector with* $0, 1$ *entries.*

These B bits per sub-channel will index 2^B regions $\mathcal{R}_{k,j} := [\tau_{k,j}, \tau_{k,j+1})$, that the g_k space is divided into using quantization thresholds $[\tau_{k,1}, \ldots, \tau_{k,2^B-1}]^T := \boldsymbol{\tau}_k$ for each sub-channel $k \in [0, \ldots, N-1]$. Notice that for brevity, we use j (rather than j_k) to index regions and thresholds; however, since the latter include also the subscript k, we should keep in mind that regions and thresholds are allowed to be different from one sub-channel to another. Further, since quantization here pertains to gains, we set $\tau_{k,0} = 0$ and $\tau_{k,2^B} = \infty$.

Our Q-CSIT based optimization will proceed in two phases: a) specification of the optimum thresholds $\{\boldsymbol{\tau}_k\}_{k=0}^{N-1}$, which can be performed off-line; and b) derivation of the optimum power and bit loading vectors $(\mathbf{m}, \mathbf{p})$ to be used online based on the Q-CSI codeword vector $\mathbf{j}$ obtained in accordance with the quantizer design phase a). We should keep in mind that after phase a) both transmitter and receiver need to know the optimal thresholds since they will use them in phase b).

A. Off-line Phase: Quantizer Design

Our goal here is to derive quantization thresholds for the random variable g_k whose pdf is known as per a2. Those are obtained upon minimization of the underlying distortion metric which will be taken to be the average power $\sum_{k=0}^{N-1} \bar{P}_k$. Therefore, the thresholds shall guarantee power savings across different channel realizations. We first note that

$$\bar{P}_k = \sum_{j=0}^{2^B-1} P_{k,j} \Pr(P_k = P_{k,j}), \tag{8}$$

where P_k is a discrete random variable taking values $\{P_{k,j}\}_{j=0}^{2^B-1}$ each denoting the power in the kth sub-channel and jth region having probability

$$\begin{aligned}\Pr(P_k = P_{k,j}) &= \Pr(g_k \in \mathcal{R}_{k,j}) \\ &:= \int_{\tau_{k,j}}^{\tau_{k,j+1}} f_{g_k}(g_k) dg_k \\ &= \exp(-\tau_{k,j}/\bar{g}_k) - \exp(-\tau_{k,j+1}/\bar{g}_k),\end{aligned} \tag{9}$$

where the last equality results from the exponential pdf in a2. This in turn reflects the dependence of the average power on the quantizing thresholds. Along with minimizing the average power, the optimal thresholds should fulfill prescribed rate and average BER constraints. Expressing the average BER per sub-channel as

$$\begin{aligned}\overline{\text{BER}}_k(\boldsymbol{\tau}_k, \{P_{k,j}\}, M_k) &= \sum_{j=0}^{2^B-1} \int_{\tau_{k,j}}^{\tau_{k,j+1}} \text{BER}_k(g_k, P_{k,j}, M_k) \\ &\times \quad f_{g_k}(g_k) dg_k\end{aligned} \tag{10}$$

(while for $j = 0$, we have $P_{k,0} = \text{BER}_k = 0$), our channel quantizer design for selecting thresholds can be formulated as follows:

$$\begin{aligned}&\min_{\tau_{k,j} \geq 0} J(\bar{\mathbf{p}}), \quad J(\bar{\mathbf{p}}) := \textstyle\sum_{k=0}^{N-1} \bar{P}_k \\ &\text{s. to} \\ &\text{C1.}\ \textstyle\sum_{k=0}^{N-1} \frac{\log_2(M_k)}{R_0} \overline{\text{BER}}_k(\boldsymbol{\tau}_k, \{P_{k,j}\}, M_k) = \text{BER}_0 \\ &\text{C2.}\ \textstyle\sum_{k=0}^{N-1} \log_2 M_k = R_0\ .\end{aligned} \tag{11}$$

As in our D-CSIT based optimization, we will first solve (11) without accounting for C2. Upon substituting the expressions for $\bar{P}_k$ and $\overline{\text{BER}}_k$ into (11) and defining $\gamma_{k,j} := \beta_k P_{k,j} + 1/\bar{g}_k$, the objective function and C1 for this relaxed problem can be written as:

$$J(\bar{\mathbf{p}}) = \sum_{k=0}^{N-1} \sum_{j=1}^{2^B-1} P_{k,j} \left(e^{-\frac{\tau_{k,j}}{\bar{g}_k}} - e^{-\frac{\tau_{k,j+1}}{\bar{g}_k}} \right). \tag{12}$$

$$\sum_{k=0}^{N-1} \frac{a \log_2 M_k}{R_0 \bar{g}_k} \sum_{j=1}^{2^B-1} \frac{e^{-\gamma_{k,j}\tau_{k,j}} - e^{-\gamma_{k,j}\tau_{k,j+1}}}{\gamma_{k,j}} = \text{BER}_0. \tag{13}$$

It can be easily shown that only locally optimal solutions can be ensured in our quantizer design due to lack of convexity. Nonetheless, the Lagrangian for minimizing (12) subject to (13) is given by

$$\begin{aligned}\mathcal{L} &= \ldots + P_{k,j-1} \int_{\tau_{k,j-1}}^{\tau_{k,j}} f_{g_k}(g_k) dg_k + P_{k,j} \int_{\tau_{k,j}}^{\tau_{k,j+1}} f_{g_k}(g_k) dg_k \\ &+ \ldots - \lambda_Q \left\{ \sum_k \frac{a \log_2(M_k)}{R_0} \left[\ldots + \int_{\tau_{k,j-1}}^{\tau_{k,j}} e^{-\beta_k g_k P_{k,j-1}} \right.\right. \\ &\left.\left. f_{g_k}(g_k) dg_k + \int_{\tau_{k,j}}^{\tau_{k,j+1}} e^{-\beta_k g_k P_{k,j}} f_{g_k}(g_k) dg_k + \ldots \right] - \text{BER}_0 \right\}\end{aligned} \tag{14}$$

where λ_Q is found after substituting into C1. Upon defining $\psi(\gamma_{k,j}, \tau_{k,j}) := (1 + \gamma_{k,j}\tau_{k,j}) e^{-\gamma_{k,j}\tau_{k,j}}$, the necessary conditions $\partial\mathcal{L}/\partial\tau_{k,j} = 0$ and $\partial\mathcal{L}/\partial P_{k,j} = 0$, respectively, yield (for $k = 0, 1, \ldots N-1$ and $j = 1, \ldots, 2^B - 1$)

$$\begin{aligned}&P_{k,j-1} - P_{k,j} - \lambda_Q \frac{a}{R_0} \log_2(M_k) \\ &\times \quad \left[e^{-\beta_k \tau_{k,j} P_{k,j}} - e^{-\beta_k \tau_{k,j} P_{k,j-1}} \right] = 0,\end{aligned} \tag{15}$$

$$e^{-\frac{\tau_{k,j}}{\bar{g}_k}} - e^{-\frac{\tau_{k,j+1}}{\bar{g}_k}} - \frac{\lambda_Q a\beta_k \log_2 M_k}{R_0 \bar{g}_k \gamma_{kj}^2} \times \quad [\psi(\gamma_{k,j},\tau_{k,j}) - \psi(\gamma_{k,j},\tau_{k,j+1})] = 0. \qquad (16)$$

Eqs. (15) and (16) can be compactly written as nonlinear functions of the powers and thresholds as $\mathcal{Z}_1(P_{k,j-1},P_{k,j},\tau_{k,j}) = 0$ and $\mathcal{Z}_2(P_{k,j},\tau_{k,j},\tau_{k,j+1}) = 0$, respectively. We solve them to obtain $\tau_{k,j}$ using the following off-line algorithm:

Q1. Sort the sub-channels in decreasing order as explained in the previous section but based on $\bar{g}_k$ and generate a table of possible $\mathbf{m}$ candidates satisfying the rate constraint. Then, execute the following loop for each entry of $\mathbf{m}$.

Q2. For $\lambda_Q = \lambda_{Q\min} : \lambda_{Q\max}$, where $\lambda_{Q\min}$ and $\lambda_{Q\max}$ values can be set from upper and lower bounds of λ_D in the D-CSIT scheme;

- For $k = 0 : N-1$
  - For $\tau_{k,1} = 0 : \tau_{\max}$, where $\tau_{\max}$ is found so that e.g., $\Pr(g_k > \tau_{\max}) = 0.01$;
    Given a $\tau_{k,1}$ value, solve (15) numerically (e.g., using the bisection method [9]) and obtain a candidate root $P_{k,1}$ (recall $P_{k,0} = 0$). With $P_{k,1}$ given, solve (16) to determine $\tau_{k,2}$. Similarly, keep alternating between (15) and (16) to eventually obtain $\{P_{k,j}\}$ and $\{\tau_{k,j}\}$ for all $j = 1,\ldots,2^B-2$. As these power and threshold values per sub-channel are found without using (16) with $j = 2^B-1$, this equation can be used to test the feasibility of the candidate solution obtained.
  - End $\tau_{k,1}$ loop.
- End k loop.

At this point, we have a list of candidate solutions $\boldsymbol{\tau}_k, \mathbf{p}$. It remains to filter these solutions and accept only those achieving the average BER constraint in (13).
End λ_Q loop.

Q3. Among all feasible $\boldsymbol{\tau}$'s, choose the one leading to the minimum $J(\bar{\mathbf{p}})$ in (12).

In step Q1, each choice of $\{M_k\}_{k=1}^{N_a}$ satisfies $N_a \leq N$ which implies that the algorithm yields the thresholds only for the N_a sub-channels that are "best on the average". Since per OFDM symbol, we may need thresholds for all sub-channels, numerical simulations have suggested a rule of thumb whereby for each j, the ratio $\tau_{k,j}/\bar{g}_k$ remains invariant for all k.

Remark 2: In addition to thresholds, the off-line algorithm yields as a byproduct $(\mathbf{m},\mathbf{p})$ loadings that either one or both of them can be used in a simpler but suboptimal instantaneous or online loading that we will discuss towards the end of the next section.

B. On-line Phase: Power and Bit Loading

With the quantizer parameters (thresholds and thus regions) fixed, each gain realization $\mathbf{g}$ is estimated and quantized at the receiver to obtain the codeword $\mathbf{j}$. Feeding back this form of instantaneous Q-CSIT, the transmitter will find on-line the pair of loadings $(\mathbf{m},\mathbf{p})$ which minimize power subject to rate, and (what we could call conditioned on $\mathbf{j}$) average BER per sub-channel. With $\gamma_k := \beta_k P_k + 1/\bar{g}_k$, the latter can be expressed as:

$$\begin{aligned}\overline{\mathrm{BER}}_{k|\mathbf{j}} &= \frac{\int_{\tau_{k,j}}^{\tau_{k,j+1}} \mathrm{BER}_k(P_k,g_k) f_{g_k}(g_k)dg_k}{\Pr(g_k \in \mathcal{R}_{k,j})} \\ &= \frac{a\left(e^{-\gamma_k\tau_{k,j}} - e^{-\gamma_k\tau_{k,j+1}}\right)}{\bar{g}_k\gamma_k\left(e^{-\tau_{k,j}/\bar{g}_k} - e^{-\tau_{k,j+1}/\bar{g}_k}\right)}. \end{aligned} \qquad (17)$$

With this conditional average BER, our optimization problem can be formulated as follows:

$$\begin{array}{l} \min_{\mathbf{m}\in\mathcal{M},\mathbf{p}(\mathbf{j})\geq\mathbf{0}} J(\mathbf{p}),\ \ J(\mathbf{p}) := \sum_{k=0}^{N-1} P_k(j) \\ \text{s. to} \\ \text{C1. } \sum_{k=0}^{N-1} \frac{\log_2 M_k}{R_0} \overline{\mathrm{BER}}_{k|\mathbf{j}}(\mathbf{m},\boldsymbol{\tau}_k,P_k,M_k|\mathbf{j}) = \mathrm{BER}_0 \\ \text{C2. } \sum_{k=0}^{N-1} \log_2 M_k = R_0\ . \end{array} \qquad (18)$$

Comparing (18) with (11), we infer that apart from the scalar in the denominator of (17), the two problems are basically identical. This scalar will alter the λ_Q multiplier, but otherwise, sorting the sub-channels according to their region indices and dropping the $\tau_{k,1}$ loop, the off-line algorithm can still be applied to solve (18). Investigating the sufficient conditions for global optimality, here requiring $\partial^2 J(\mathbf{p})/\partial P_k^2 \geq 0$ and $\partial^2 \overline{\mathrm{BER}}_{|\mathbf{j}}/\partial P_k^2 \geq 0$, the first one can be readily verified while the latter yields $\frac{\partial^2 \overline{\mathrm{BER}}_{|\mathbf{j}}}{\partial P_k^2} = \frac{a\log_2 M_k}{R_0\left(e^{-\tau_{k,j}/\bar{g}_k} - e^{-\tau_{k,j+1}/\bar{g}_k}\right)} \int_{\tau_{k,j}}^{\tau_{k,j+1}} (\beta_k g_k)^2 e^{-\gamma_k g_k} dg_k \geq 0$ where the last inequality follows since $\tau_{k,j} < \tau_{k,j+1}$ and the integrand is non-negative.

In the spirit of Remark 2, the transmitter in the on-line phase can operate in any of the following modes (listed in decreasing power efficiency but also in decreasing complexity order):

Tx A : select optimally the set of active sub-carriers $\mathcal{N}_a$, bit loading $\mathbf{m}$, and the value of the Lagrange multiplier, λ, to attain the global optimum $\mathbf{p}$.
Tx B : select optimally $\mathcal{N}_a$ and λ while using a fixed $\mathbf{m}$ (no constellation adaptation).
Tx C : select optimally $\mathcal{N}_a$, but use fixed values for $\mathbf{m}$ and λ.
Tx D : select optimally λ, but use always the same $\mathcal{N}_a$ and $\mathbf{m}$.

Having considered D-CSIT and Q-CSIT, we look next into the possibility of exploiting only statistical knowledge of the channel. The resultant S-CSIT based OFDM is the least power efficient but also incurs the lowest complexity and feedback overhead.

V. Optimization Based on S-CSIT

Based on the channel pdf (which in our case depends only on $\bar{g}_k$), our objective in this section is to determine the power minimizing *fixed* pair of loadings $(\mathbf{m},\mathbf{p})$. The problem can be formulated as:

$$\begin{array}{l} \min_{\mathbf{m}\in\mathcal{M},\ \bar{\mathbf{p}}(\bar{\mathbf{g}})\geq\mathbf{0}} J(\bar{\mathbf{p}}),\quad J(\bar{\mathbf{p}}) := \sum_{k=0}^{N-1} \bar{P}_k(\bar{g}_k) \\ \text{s. to} \\ \text{C1. } \sum_{k=0}^{N-1} \frac{\log_2 M_k}{R_0} \overline{\mathrm{BER}}_k(P_k,M_k,\bar{g}_k) = \mathrm{BER}_0 \\ \text{C2. } \sum_{k=0}^{N-1} \log_2 M_k = R_0, \end{array} \qquad (19)$$

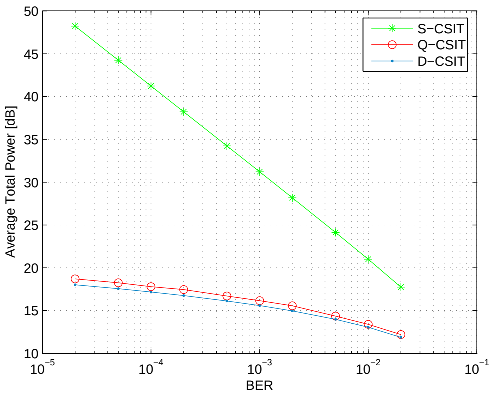

Fig. 2. Comparison of systems with different types of CSIT and variable BER constraints ($N = 64$, $R_0 = 25$, $B = 2$).

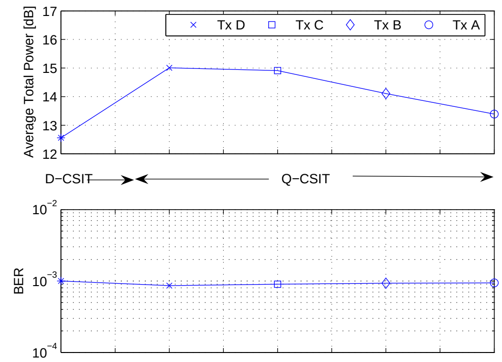

Fig. 3. Comparison of D-CSIT and Q-CSIT schemes ($N = 64$, $\mathrm{BER}_0 = 10^{-3}$, $R_0 = 20$, $B = 2$).

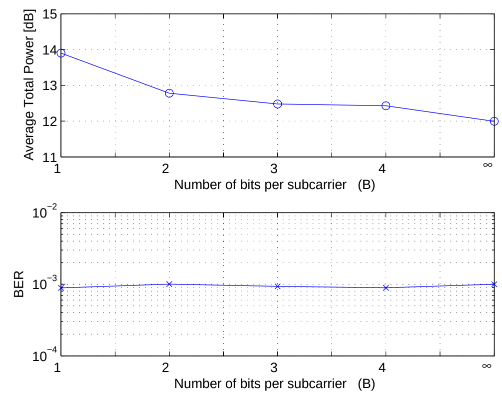

Fig. 4. Effect of the number of feedback bits when using the Q-CSIT based scheme ($N = 64$, $R_0 = 25$).

where

$$\begin{aligned}\overline{\mathrm{BER}}_k(P_k, M_k, \bar{g}_k) &= \int_0^{\infty} e^{-\beta_k P_k g_k} f_{g_k}(g_k) dg_k \\ &= \frac{a}{1+\beta_k P_k \bar{g}_k}. \end{aligned} \quad (20)$$

Removing C2 as before, the KKT conditions on the relaxed problem yield for $k \in [0, \dots, N-1]$,

$$\bar{P}_k = \left[\lambda_S \sqrt{\frac{a \log_2 M_k}{R_0 \beta_k \bar{g}_k}} - \frac{1}{\beta_k \bar{g}_k}\right]^+, \quad (21)$$

where $\lambda_S = \frac{1}{\mathrm{BER}_0} \sum_{k=0}^{K-1} \sqrt{\frac{a \log_2 M_k}{R_0 \beta_k \bar{g}_k}}$. The sufficient conditions for optimality, $\frac{\partial^2 \bar{P}}{\partial \bar{P}_k^2} \geq 0$ and $\frac{\partial^2 \overline{\mathrm{BER}}}{\partial \bar{P}_k^2} \geq 0$, are easily shown to hold true, thus ensuring global optimality. As discussed earlier, (21) must be calculated for all sorted candidates $\mathbf{m}$ that meet the R_0 constraint, selecting as final solution the one which requires the least power.

VI. Numerical Examples

To numerically test our power-efficient designs, we considered an adaptive OFDM system with $N = 64$ sub-carriers and $B = 2$ feedback bits per sub-carrier (when Q-CSIT is utilized) with average gains $\{\bar{g}_k\}_{k=0}^{N-1}$ allowed to vary over a 10dB range, and prescribed to achieve $\mathrm{BER}_0 = 10^{-3}$ at rate $R_0 = 25$ bits (unless otherwise specified).

Test Case 1 *(D-CSIT vs. Q-CSIT vs. S-CSIT in Power Efficiency and BER)* : For variable BER specifications, Figure 2 compares our preferred Q-CSIT scheme relative to the D-CSIT and S-CSIT benchmarks. We observe that Q-CSIT needs only around 1dB more power than D-CSIT and exhibits an average power gain of approximately 8, 15, and 24dB for $\mathrm{BER}_0 = 10^{-2}$, 10^{-3}, and 10^{-4}, respectively over the S-CSIT design. Compared to the no CSIT case, this gain almost doubles since the S-CSIT scheme has itself a power gain in the range of 15 to 25dB. Surprisingly, we also observe the almost-uniform reduction in power savings of the Q-CSIT based design compared to the D-CSIT based one across a wide range of BER values.

Test Case 2 *(D-CSIT vs. Q-CSIT)* : Figure 3 quantifies the power loss when relying on Q-CSIT rather than D-CSIT for the different adaptive transmitters TxA-D. We observe that all four strategies TxA-D perform within 2 dB with respect to each other. In addition, the fully adaptive transmission (Tx-A) shows near-optimum power efficiency almost 1 dB away from the optimal D-CSIT benchmark.

Test Case 3 *(Number of Quantization Regions)* : We finally investigated the effect of the number of feedback bits in the Q-CSIT scheme. Figure 4 demonstrates that power gains increase markedly when B increases from 1 to 2 bits, while a diminishing relative gain is expected for each additional increase in B. It also shows that having $B = 1$ entails only 2dB in power loss compared to the D-CSIT case which corresponds to $B = \infty$.

VII. Conclusions

Under prescribed rate and error probability constraints, we optimized power-efficiency of wireless OFDM by relying on various forms of CSIT. Emphasis was placed on Q-CSIT which can become readily available because the receiver needs to feed back to the transmitter only a limited number of bits indexing the quantization region which the underlying channel falls into.

To choose a judicious form of Q-CSIT, we designed off-line optimal channel quantizers that rely on channel statistics (mean) to select quantization thresholds across OFDM sub-channels. We then designed an online optimum algorithm that allocates modulation and power levels across the sub-channels. The resultant optimized transceivers hold great potential since they were shown to enjoy about 15dB power savings relative to S-CSIT (and twice as much when compared to OFDM with no-CSIT available), while staying within 1dB from D-CSIT benchmark designs that are more suitable for slow fading wireless or wireline channels. It was also shown that having two feedback bits per sub-channel captures most of the gain loss due to quantization.

The promise power-efficient OFDM holds in the single-antenna point-to-point scenario considered here, motivates future research to explore its potential for power savings when multi-carrier modulation is used over multi-user and (possibly distributed) multi-antenna links in the context of ad hoc and wireless sensor networks.

References

[1] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley series in Telecommunications, 1991.

[2] F. F. Digham and M. S. Alouini, "Diversity combining with discrete power loading over fading channels," in *Proc. of Wireless Comm. and Networking Conf.*, Atlanta, GA, pp. 328–332, March 2004.

[3] F. F. Digham and M. -S. Alouini, "Variable-Rate Variable-Power Hybrid M-FSK M-QAM for Fading Channels," in *Proc. of IEEE Veh. Technol. Conf.*, Orlando, FL, October 2003.

[4] L. Goldfeld, V. Lyandres, and D. Wulich, "Minimum BER power loading for OFDM in fading channel," *IEEE Trans. on Commun.*, vol. 49, pp. 14–18, January 2001.

[5] A. J. Goldsmith and S. -G. Chua, "Variable-rate variable-power M-QAM for fading channels," *IEEE Trans. on Commun.*, vol. 45, pp. 1218–1230, October 1997.

[6] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*. Holden-Day, Inc., 1986.

[7] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Commun. Lett.*, vol. 48, pp. 23–27, January 2000.

[8] D. J. Love and R. W. Heath Jr., "Limited feedback power loading for OFDM," *Proc. of Military Comm. Conf.*, Monterey, CA, pp. 71–77, Oct. 2004.

[9] T. K. Moon and W. C. Stirling, *Mathematical Methods and Algorithms for Signal Processing*. Upper Saddle River, New Jersey: Prentice Hall Publishers, 2000.

[10] R. Nilsson, O. Edfors, M. Sandell, and P. O. Boerjesson, "An analysis of two-dimensional pilot-symbol assisted modulation for OFDM," in *Proc. of Intl. Conf. On Personal Wireless Comm.,* Bombay, India, pp. 71–74, Dec. 1997.

[11] L. Piazzo, "Fast algorithm for power and bit allocation in OFDM systems," *IEE Elect. Lett.*, vol. 35, pp. 2173 – 2174, Dec. 1999.

[12] H. Qian, C. Xiao, N. Chen, and G. T. Zhou, "Dynamic selected mapping for OFDM," in *Proc. of Intl. Conf. on Acoustics, Speech, and Signal Processing,* Philadelphia, PA, pp. 325–328, March 2005.

[13] Z. Wang and G. B. Giannakis, "Wireless Multicarrier Communications: Where Fourier Meets Shannon," *IEEE Signal Processing Magazine*, Vol. 17, No. 3, pp. 29-48, May 2000.

[14] Y. Yao and G. B. Giannakis, "Rate-Maximizing Power Allocation in OFDM Based on Partial Channel Knoweledge," *IEEE Trans. on Wireless Communications*, vol. 4, no. 3, pp. 1073-1083, May 2005.