ABSTRACT

Transmit-power control is a critical task in cognitive radio (CR) networks. In the present contribution, adherence to hierarchies between primary and secondary users in a peer-to-peer CR network is enabled through distributed power control. Hierarchies are effected by imposing minimum and maximum bounds on a quality-of-service (QoS) metric, such as communication rate. These bounds translate to signal-to-interference-plus-noise ratio (SINR) constraints. Furthermore, a utility function captures each user’s satisfaction with the received SINR. The novel power control strategy maximizes the total utility while respecting individual SINR constraints—a task cast as a convex optimization problem under a suitable relaxation. Sufficient conditions, realistic for practical CR networks, are provided to obtain the optimal power allocation from the solution of the relaxed problem. Finally, a low-overhead distributed algorithm for optimal power control is developed, and tested against competing alternatives via simulations.

Index Terms—Cognitive radios, distributed algorithms, optimization methods, power control, QoS constraints

1. INTRODUCTION

Cognitive radio is an emerging technology promising efficient spectrum utilization by dynamically adapting to the conditions of the operating environment [1]. In a CR network, primary users or licensees coexist with secondary and/or unlicensed users or lessees, who have limited access to network resources [1, §49]. Such a regulated access can be realized by bounding the maximum level of a commodity a user can receive, which may be communication rate (as in [2], [3]), bit error rate (BER), or any other QoS figure. Such bounds lead in turn to heterogeneous QoS requirements of the CR users.

Adjusting transmission power [1, §27] offers the potential to satisfy these requirements. The challenge however, is to mitigate co-channel interference, which is intimately coupled with individual power control decisions. This paper deals with a utility-based approach to power control in peer-to-peer CR networks, where the satisfaction of each user with the received QoS level is captured by a utility function, which depends on the received SINR; see also [4], [5], [3] and references therein.

So far, two sub-optimal algorithms have been reported for distributed power control in CR networks with diverse QoS constraints [3]. The contribution of the present work is two-fold: (i) optimal power control is obtained using convex optimization; and (ii) a practical distributed algorithm for optimal power control is developed to account for heterogeneous QoS requirements tailored to the CR paradigm (not considered in other works on utility-based power control [4], [5], [6]).

The remainder of the paper is organized as follows. The power control problem is stated in Sec. 2 and solved in Sec. 3. The resultant distributed algorithm is the subject of Sec. 4, while Sec. 5 presents simulations and Sec. 6 conclusions.

2. PROBLEM STATEMENT

Consider a wireless peer-to-peer network with a set of \( \mathcal{M} := \{1, \ldots, M\} \) links, as in [4], [3], where each link \( i \in \mathcal{M} \) entails a user with a dedicated transmitter (Tx) \( _i \) wishing to communicate with a corresponding receiver (Rx). All links are assumed sharing the same frequency band (referred to as single-channel in [4]), e.g., in CDMA. Let \( h_{ij} \) denote the channel gain from Tx \(_i \) to Rx \(_j \) (assumed static); \( n_i \) the noise power at Rx \(_i \); and \( p_i \) the transmission power of Tx \(_i \). Suppose that Tx \(_i \) can transmit with at least \( P_{i}^{\text{min}} \) and at most \( P_{i}^{\text{max}} \) power budget, i.e., \( p_i \in \mathcal{P}_i := [P_{i}^{\text{min}}, P_{i}^{\text{max}}] \). The received SINR \( \gamma_i \) at Rx \(_i \) is a function of the powers \( p := (p_1, \ldots, p_M) \) and is given by

\[
\gamma_i := \frac{h_{ii}p_i}{n_i + \sum_{k\neq i} h_{ki}p_k}. \tag{1}
\]

Each user link \( i \in \mathcal{M} \) adopts a utility function \( u_i(\gamma_i) \) that reflects the received QoS level. The utilities are selected concave, strictly increasing and twice continuously differentiable over \((0, \infty)\). We will focus on two important utilities (with \( w_i > 0 \) and \( \alpha < 0 \)):

\[
u_i(\gamma_i) = w_i \ln \gamma_i \quad \text{or} \quad u_i(\gamma_i) = w_i \gamma_i^{\alpha - 1} \gamma_i^\alpha. \tag{2}\]

These utilities satisfy (with \( \alpha = 0 \) if \( u_i(\gamma_i) = w_i \ln \gamma_i \))

\[
C_i(\gamma_i) := \frac{\gamma_i u''_i(\gamma_i)}{u'_i(\gamma_i)} = 1 - \alpha, \quad \forall \gamma_i > 0. \tag{3}
\]
The ratio $C_i(\gamma_i)$ is instrumental in ensuring convexity of the power control problem; see [5, Ch.5], [4] and references therein. In future submissions, we will consider general utilities for which $C_i(\gamma_i)$ is allowed to vary with $\gamma_i$. Note that for the utilities in (2) it is also necessary to have $P^\text{min}_{i} > 0$. This imposes no practical restriction, since setting $P^\text{min}_i$ to a very small value effectively amounts to no transmission.

In the framework of CR networks, the maximum level of individual user commodities is bounded. To achieve this, the focus here is on commodities that are one-to-one functions of $\gamma_i$. These include rate $\log(1 + \gamma_i)$ [2], [3], BER, or each user’s utility function $u_i(\gamma_i)$. A bound on the maximum level of the commodity readily maps to an SINR constraint $\gamma_i \leq \gamma^\text{max}_i$. Moreover, such a constraint is pertinent when further increase in $\gamma_i$ cannot effectively increase the user’s utility, as e.g., in fixed-rate services [3]. Also note that QoS guarantees for primary (or even secondary) users can be provided through a minimum SINR constraint $\gamma_i \geq \gamma^\text{min}_i$.

Power control in networks with heterogeneous QoS constraints amounts to selecting powers $p_i$ that maximize the total utility of the users, $\sum_{i=1}^{M} u_i(\gamma_i)$, while respecting the individual SINR requirements; i.e., the goal is

$$\max_{\{p_i \in P_i, \forall i \in M\}} \sum_{i=1}^{M} u_i(\gamma_i)$$

subject to $\gamma_i^{\text{min}} \leq \gamma_i \leq \gamma_i^{\text{max}}$, $\forall i \in M$. (4a)

In general, problem (4) is non-convex in $p_i$; certain instances of (4) though, are known to be equivalent to convex problems. Specifically, in the absence of (4b) and for a general class of utility functions which includes (2), problem (4a) can be written as a convex optimization problem under the transformation $p_i = e^{y_i}$ [5]; see also [4]. (From [4] and [5] it can be inferred that minimum SINR constraints $\gamma_i \geq \gamma_i^{\text{min}}$ can also be handled, although this is not explicitly treated.) Finally, under minimum SINR constraints only $\gamma_i \geq \gamma_i^{\text{min}}$, problem (4) can also be written as a geometric program (GP), if $u_i(\gamma_i) = w_i \ln \gamma_i$; and as a generalized GP, if $u_i(\gamma_i) = w_i e^{-\gamma_i}$ [6]. (In the area of GP, the transformation $p_i = e^{y_i}$ is also standard [6].)

None of the aforementioned works can accommodate the maximum SINR constraint $\gamma_i \leq \gamma_i^{\text{max}}$, pertinent to CR networks. It is the present paper’s contribution to tackle the solution of (4), through a suitable relaxation, elaborated below.

### 3. OPTIMAL POWER CONTROL

Let $q_i$ be an auxiliary variable, associated with link $i$, upperbounding the true interference-plus-noise denominator in (1). Collecting all $q_i$’s in a vector $q := (q_1, \ldots, q_M)$, consider the following relaxed version of (4):

$$\max_{\{p_i \in P_i, \forall i \in M\}} \sum_{i=1}^{M} u_i(h_{ii} p_i q_i^{-1})$$

subject to $\gamma_i^{\text{min}} \leq h_{ii} p_i q_i^{-1} \leq \gamma_i^{\text{max}}$, $\forall i \in M$ (5b)

$$q_i \geq n_i + \sum_{k \neq i} h_{ki} p_k, \quad \forall i \in M$$

where $\mathbb{R}_+$ are the positive reals. Clearly, if $(5c)$ are equality constraints, then problems (4) and (5) would be equivalent. Even though (5) is not jointly convex in $p_i$ and $q_i$, it will be possible to transform it into an equivalent convex optimization problem.

To this end, apply the one-to-one change of variables $p_i = e^{y_i}, q_i = e^{z_i}$. Then the power constraints in (5a) map to $P^\text{min}_i, e^{-y_i} \leq 1$ and $(P^\text{max}_i)^{-1} e^{-y_i} \leq 1$; the SINR constraints become $\gamma_i^{\text{min}} h_{ii}^{-1} e^{y_i - z_i} \leq 1$, $\gamma_i^{\text{max}} h_{ii} e^{y_i - z_i} \leq 1$; and those in (5c) translate to $n_i e^{z_i} + \sum_{k \neq i} h_{ki} e^{y_k - z_i} \leq 1$. The transformed constraints are convex in $y := (y_1, \ldots, y_M)$ and $z := (z_1, \ldots, z_M)$ since all left-hand sides are compositions of nonnegative sum of exponentials (which are convex functions) with affine mappings [7, Sec. 3.2].

What remains to show is that the objective in (5a) is concave in $y_i, z_i$. Since it is a nonnegative sum of $u_i(e^{y_i - z_i + \ln h_{ii}})$ terms, it suffices for $u_i(e^{z_i})$ to be concave in the scalar $x \in \mathbb{R}$, i.e., that $d^2u_i(e^{z_i}) \leq 0 \iff C_i(z) = -\xi e^{\xi(z)} \geq 1 \, (\xi = e^{z_i})$.

Now define matrix $A := [a_{ij}]$ with $a_{ij} = 0 \forall i \in M$ and $a_{ij} = h_{ji} / h_{ii} \forall j \neq i$. (It is common to collect channels $h_{ij}$ in such a matrix; see e.g., [5].) The following result asserts that under mild conditions the solution of (5) also solves (4).

Proposition 1 Assume that: (a1) problem (4) is feasible; (a2) utilities $u_i(\gamma_i)$ are continuous and strictly increasing; (a3) matrix $A$ is irreducible; (a4) there is no power vector $p$ with $p_i \in P_i, \forall i \in M$ s.t. $\gamma_i = \gamma_i^{\text{max}}, \forall i \in M$; and (a5) the constraint $P^\text{min}_i$ is sufficiently small s.t. $h_{ii} P^\text{min}_i / n_i < \gamma_i^{\text{max}}$. If $p^*, q^*$ solve problem (5), then (5c) holds as equality at $p^*, q^*$; i.e.,

$$q^*_i = n_i + \sum_{k \neq i} h_{ki} p^*_k, \quad \forall i \in M.$$  

It is worth stressing that Prop. 1 holds for any strictly increasing utility, not only the ones in (2). Note further that the assumption $h_{ii} P^\text{min}_i / n_i < \gamma_i^{\text{max}}$ is innocuous, since $P^\text{min}_i$ is selected so small that it amounts to no transmission. Moreover, the assumption on the irreducibility [5, Def. A.21] of $A$ is common in power control problems; see e.g., [5, Sec. 5.5].

The non-achievability condition on the SINRs $\gamma_i^{\text{max}}$ within the power constraints for all users is slightly more restrictive and should be checked before solving (5). It is important to remark that if the SINRs $\gamma_i^{\text{max}}$ are achievable for all users, then the optimal total utility will be $\sum_{i=1}^{M} u_i(\gamma_i^{\text{max}})$ and no further optimization is needed. If not though, the solution of (4) will yield the optimal power allocation.

To check this, we rely on a classical power control algorithm for given SINR requirements [8]. Specifically, consider the iteration $p(t + 1) = I(p(t))$, called standard power control algorithm (SPCA), where $I(p) := [I_1(p), \ldots, I_M(p)]$ with

$$I_i(p) := \min \left\{ P^\text{max}_i, \gamma_i^{\text{max}} - \frac{1}{h_{ii}} (n_i + \sum_{k \neq i} h_{ki} p_k) \right\}.$$  

From [8, Cor. 1] it follows that the algorithm converges, and upon convergence, all users will have $\gamma_i = \gamma_i^{\text{max}}$ if and only if this is feasible under the constraint $p_i \leq P^\text{max}_i \forall i \in M$ (and then $p_i \geq P^\text{min}_i \forall i \in M$ due to $h_{ii} P^\text{min}_i / n_i < \gamma_i^{\text{max}}$).

Proofs are omitted due to space limitations.
otherwise, at least one user will have $\gamma_i < \gamma_{i,\max}$. Furthermore, the SPCA can be implemented in a distributed fashion, without any exchange of information among users [8, Sec. VI].

Proposition 1 allows optimizing the power allocation (when not all $\gamma_{i,\max}$ are achievable) by solving the Karush-Kuhn-Tucker (KKT) conditions [7, Sec. 5.5.3] of the convex equivalent of (5). This is the theme of the ensuing subsection.

3.1. Solution of the KKT conditions

Let $\lambda_i^t, \lambda_i^{\ell}, \mu_i$ denote Lagrange multipliers corresponding to min and max SINR constraints (5b) and (5c), respectively. The Lagrangian of the convex equivalent of (5) is

$$L(y, z, \lambda^t, \lambda, \mu) := \sum_i \mu_i \left[ e^{-z_i} \left( n_i + \sum_{k \neq i} h_{ki} e^{y_k} \right) - 1 \right] - \sum_i u_i \left( h_{i,\max} z_i - z_i \right) + \sum_{i,j} \lambda_i^{\ell} \left( \gamma_{i,\max} h_{ij} e^{-z_{ij}} - 1 \right) + \sum_i \lambda_i^t \left( \gamma_{i,\min} h_{i,\max} - e^{-z_i} - 1 \right).$$

The Lagrangian is separable in $z_i$; hence, the $z_i$ which minimizes the Lagrangian can be obtained given $y, \lambda^t, \lambda^{\ell}, \mu_i$ for each $i \in M$ by taking $\partial L / \partial z_i = 0$. The latter yields:

$$u_i' \left( e^{y_i} / e^{z_i} \right) - \mu_i \left( n_i + \sum_{k \neq i} h_{ki} e^{y_k} / e^{z_i} \right) + e^{z_i} \lambda_i^{\ell} \gamma_{i,\min} (h_{i,\max} h_{i,\max} - 1) - \lambda_i^t \gamma_{i,\max} = 0$$

Eq. (7) can be solved for $e^{z_i}$ as a function of $y, \lambda^t, \lambda^{\ell}, \mu_i$. In fact, all quantities needed for solving (7) are known locally at Tx, or Rx$_i$. Specifically, these are the local Lagrange multipliers $\lambda_i, \mu_i$, the received power $h_{i,\max} e^{y_i}$, and the measured SINR, $h_{i,\max} e^{y_i} / (n_i + \sum_{k \neq i} h_{ki} e^{y_k})$.

Since optimal powers $y^*$ and optimal Lagrange multipliers $\lambda^{*,\ell}, \lambda^{*,t}, \mu^*$ cannot be obtained in closed form, namely by solving $\partial L / \partial y_i = 0$ directly, an iterative algorithm is needed. The exact form of a Lagrangian gradient-based algorithm and its distributed implementation are presented next.

4. DISTRIBUTED ALGORITHM

In this section, we present a distributed algorithm to solve the convex equivalent of (5). Let $\bar{z}_i$ denote the optimal value of $z_i$ as a function of $y, \lambda^t, \lambda^{\ell}, \mu_i$, obtained locally from (7). Then at any $y, \lambda^t, \lambda^{\ell}, \mu$ (and corresponding $\bar{z}$) we have

$$\frac{\partial L}{\partial y_i} = -u_i' \left( h_{i,\max} e^{y_i} / e^{z_i} \right) h_{i,\max} e^{y_i} / e^{z_i} + e^{z_i} \lambda_i^{\ell} \gamma_{i,\min} (h_{i,\max} h_{i,\max} - 1) - \lambda_i^t \gamma_{i,\max}.$$ 

Further, define a beacon variable $b_{ij} := \mu_i e^{z_{ij}}$ and observe that the variables $b_{ij}$ as well as the channels $h_{ij}$ are the only non-local (to Tx$_i$ or Rx$_j$) quantities that $\partial L / \partial y_i$ depends on.

Now let $Y_{i,\min} := \ln \gamma_{i,\min}, Y_{i,\max} := \ln \gamma_{i,\max}$, and $[Y_{i,\min}, Y_{i,\max}]$ the projection onto $[\gamma_{i,\min}, \gamma_{i,\max}]$; and $[\gamma_{i,\min}, \gamma_{i,\max}]$ onto the non-negative reals. Then the optimal powers $y^*$ and Lagrange multipliers $\lambda^{*,\ell}, \lambda^{*,t}, \mu^*$ can be obtained by gradient projection iterations (indexed by $t$) with constant stepsize $\beta$.

$$y_i(t + 1) = \left[ y_i(t) - \beta \frac{\partial L}{\partial y_i} \right]_{y_i = Y_{i,\min}}^{Y_{i,\max}}$$

$$\lambda_i^{\ell}(t + 1) = \left[ \lambda_i^{\ell}(t) + \beta \left( e^{z_i(t)} \gamma_{i,\min} - 1 \right) / h_{i,\max} e^{y_i(t)} \right]^{+}$$

$$\lambda_i^t(t + 1) = \left[ \lambda_i^t(t) + \beta \left( h_{i,\max} e^{y_i(t)} - 1 \right) / h_{i,\max} e^{\gamma_{i,\min}} \right]^{+}$$

$$\mu_i(t + 1) = \left[ \mu_i(t) + \beta \left( n_i + \sum_{k \neq i} h_{ki} e^{y_k(t)} / e^{z_i(t)} - 1 \right) \right]^{+}.$$

The updates for $y_i, \lambda_i^{\ell}, \lambda_i^t, \mu_i$ take place at the transmitter of link $i \in M$. In [4], this is possible provided that Tx$_i$ knows: (i) the SINR $h_{i,\max} e^{y_i} / (n_i + \sum_{k \neq i} h_{ki} e^{y_k})$ at every timeslot $t$ and the channel $h_{i,\max}$ (through feedback from Rx$_i$); (ii) the channels $h_{ij}$ to Rx$_j$ (by reciprocity if Rx$_j$ transmits a training signal); and (iii) the beacon variables $b_{ij}$. Note that each $b_{ij}$ is known at Tx$_j$, so every transmitter must broadcast its beacon variable to all other transmitters. Nevertheless, it is only a scalar quantity that must be broadcasted. This type of message passing in utility-based power control is also used in [4], [6, Ch. 3], while a simpler scheme is advocated in [5, Sec. 6.5.4].

We contend that the updates (9)–(12) can be implemented in a distributed fashion. Indeed, observe that the updates (10)–(12) need only quantities locally available at each Tx$_i$. Specifically, $z_i$ can be evaluated at Tx$_i$, if the current SINR and channel $h_{i,\max}$ fed back from Rx$_i$. Similarly, the interference-plus-noise $n_i + \sum_{k \neq i} h_{ki} e^{y_k}$ depends only on the current SINR, $h_{i,\max}$, and power $e^{y_i}$. For the evaluation of $\partial L / \partial y_i$ in (9), the variables $b_{ij}$ need to be acquired at Tx$_j$ as described earlier in (iii), and the channels $h_{ij}$ are available by assumption (ii). All other quantities involved in $\partial L / \partial y_i$ are known at Tx$_i$, by (i).
Table 1. Coordinates of 8 Tx-Rx pairs (shown in 2 columns).

<table>
<thead>
<tr>
<th>Pair</th>
<th>Coordinates $Tx_i$, $Rx_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(4.80,5.15);(4.92,3.67)$</td>
</tr>
<tr>
<td>2</td>
<td>$(5.61,6.06);(6.11,7.51)$</td>
</tr>
<tr>
<td>3</td>
<td>$(6.16,9.67);(4.70,10.93)$</td>
</tr>
<tr>
<td>4</td>
<td>$(6.62,8.22);(5.17,9.39)$</td>
</tr>
<tr>
<td>5</td>
<td>$(6.17,3.18);(6.95,4.40)$</td>
</tr>
<tr>
<td>6</td>
<td>$(6.85,5.88);(8.07,6.70)$</td>
</tr>
<tr>
<td>7</td>
<td>$(5.10,1.30);(4.45,0.12)$</td>
</tr>
<tr>
<td>8</td>
<td>$(7.14,2.54);(5.83,1.05)$</td>
</tr>
</tbody>
</table>

Table 2. Total utility (top) and SINR per user (bottom) achieved by different algorithms.

<table>
<thead>
<tr>
<th>$\gamma_i$</th>
<th>Lagrangian</th>
<th>QoS-ps-DSA</th>
<th>QoSe-DSA</th>
<th>ADP</th>
<th>SPCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>32.43</td>
<td>1.4e-07</td>
<td>7.1e-08</td>
<td>81</td>
<td>70.4</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>43.3</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>191.1</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>6.2</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>32.9</td>
<td>52.5</td>
<td>91.3</td>
<td>55.2</td>
<td>81.4</td>
</tr>
<tr>
<td>$\gamma_6$</td>
<td>786.2</td>
<td>655.3</td>
<td>904.7</td>
<td>443</td>
<td>734.2</td>
</tr>
<tr>
<td>$\gamma_7$</td>
<td>140</td>
<td>140</td>
<td>140</td>
<td>544.9</td>
<td>140</td>
</tr>
<tr>
<td>$\gamma_8$</td>
<td>30</td>
<td>32.2</td>
<td>25.6</td>
<td>7.5</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Fig. 1. Convergence of powers and Lagrange multipliers.

6. Conclusions

The present work tackled several aspects of the power control problem in peer-to-peer CR networks. The hierarchy between primary and secondary users was manifested through appropriate minimum and maximum SINR constraints, while a utility function was employed as a QoS indicator for each user. The optimal power control was obtained by considering a relaxed version of the original problem, which was shown equivalent to a convex problem; interestingly, a solution of the original problem could be recovered from the relaxed one under mild assumptions. Finally, a distributed algorithm for optimal power control requiring exchange of a scalar quantity was developed.2

7. References


2The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Lab. or the U. S. Government.