Dynamic Resource Management for Cognitive Radios using Limited-Rate Feedback

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***Abstract*—Tailored for the emerging class of cognitive radio networks comprising primary and secondary wireless users, the present paper deals with dynamic allocation of sub-carriers, rate and power resources based on channel state information (CSI) for orthogonal frequency-division multiple access (OFDMA). Users rely on adaptive modulation, coding and power modes that they select in accordance with the limited-rate feedback they receive from the access point. The access point uses CSI to maximize a generic concave utility of the average rates in the network while adhering to rate and power constraints imposed on the primary and secondary users to respect cognitive radio related hierarchies. When the channel distribution is available, optimum dual prices are found to optimally allocate resources across users dynamically per channel realization. In addition, a simple yet optimal on-line algorithm that does not require knowledge of the channel distribution and iteratively computes the dual prices per channel realization is developed using a stochastic dual approach. Analysis of the computational and feedback overhead along with simulations assessing the performance of the novel algorithms are also provided.**

Keywords: Cognitive radios, dynamic resource management,scheduling, non-linear convex optimization, dual formulation, adaptive signal processing, quantization.

I. Introduction

The proliferation of wireless services along with the perceived spectrum under-utilization have motivated recent research on dynamic spectrum management and wireless cognitive radios (CRs). CRs capable of sensing the spectrum, allocating radio resources and accessing the system bandwidth dynamically, as well as using available stimuli to track down their environment. A number of challenges arise with such dynamic and hierarchical means of accessing the spectrum [14]. As CR users typically communicate in an opportunistic manner, they should be capable of sensing the spectrum over a wide range of frequencies and perform online dynamic scheduling and allocation of resources to improve bandwidth utilization. Accordingly, CR users must be capable of adapting their transmission and reception parameters to the intended dynamically changing channel while respecting possible hierarchies and adhering to power constraints and diverse quality of service (QoS) requirements. Adapting to the environment and the channel conditions constitutes one of the major tasks in CR research and development, and is the main focus of this paper.

The merits of adaptive schemes which exploit knowledge of *perfect* (P-) instantaneous channel state information (CSI) and channel statistics to optimally allocate the transmit resources in wireless systems are well documented; see e.g., [13] and [10, Chap. 9] for point-to-point, or, [33], [20] and [36] for multi-user links. Since the assumptions of perfect knowledge of the instantaneous CSI or the channel statistics may be unrealistic for many practical systems, recent research has focused on adaptive schemes that: (i) rely on instantaneous *quantized* (Q-) CSI that can be pragmatically acquired via a limited-rate feedback link; see e.g., [24], [21], [25]; and (ii) do not require channel statistics but allocate resources based on *stochastic* approximation algorithms (these can be viewed as "intelligent" least mean-square (LMS) type schemes which learn the unavailable information on-the-fly); see [30], [9] and [35] for a recent review. Application of such adaptive schemes has been recently investigated to manage resources in CR based on different approaches: utility optimization [16], [37], [26] and economic bid [2]; game theory [15], [38]; and multi-agent schemes [17], to name a few; see also [1] and [28] for surveys.

The present paper investigates resource allocation (RA) based on Q-CSI for CR operating over fading channels, with known or unknown channel statistics. The focus is on a CR, where co-existing primary and secondary users [28] rely on orthogonal frequency multiple access[1] (OFDMA). For such a scenario, the access point relies on the current CSI, channel statistics, and user specifications to optimally allocate resources and notify users about the optimal schedule through a feedback channel. This allows users to adapt their transmissions (power, rate, and subchannel) accordingly. Channel-adaptive transmissions mitigate the adverse effects of fading, and further exploit the diversity provided by the channel. The

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[1] In principle, any other orthogonal basis can be used as a set of transmit waveforms.

main contributions are:

- Channel-adaptive resource (power, rate, subcarrier) allocation is obtained as the solution of a constrained optimization problem, which naturally takes into account different user priorities, specific utility functions, individual QoS requirements, and physical layer parameters, e.g., channel statistics. The resultant optimum dynamic resource allocation depends only on the current channel realization, and dual variables that can be readily interpreted as user-specific prices.
- While the resource allocation is found in closed-form, the user-specific prices capture the differences among users in terms of priority, QoS, as well as average channel conditions. Optimal prices are acquired for two different scenarios: (i) when channel statistics are known, allocation relies on convex optimization tools [5] and amounts to an iterative algorithm that converges to the optimum value of the dual prices; and (ii) when channel statistics are not known, an adaptive stochastic algorithm is developed capable of learning the intended channels on-the-fly, and converging in probability to the optimal solution.
- Computational and feedback overhead is assessed for both time division duplex (TDD) and frequency division duplex (FDD) CRs, and found to be affordable for most practical systems. Further reduction of the feedback requirements is accomplished through channel quantization, which yields reduced-size optimal power and rate codebooks.

Recent research has investigated distributed RA for wireless CR with primary and secondary users to optimize the sum-capacity for CDMA [31] and OFDMA systems [3] based on P-CSI. Different from these works, the present paper considers: (i) maximization of a generic utility objective entailing practical adaptive power, modulation, and coding schemes; (ii) an access point collecting the CSI and performing the optimal resource allocation; and (iii) instead of P-CSI, practically affordable Q-CSI via a finite-rate feedback link between the access point and the users. The access point and a feedback channel have been consistently recognized as facilitators of both sensing and intelligence [14], tasks of paramount importance for CRs.

Another problem that during the last years has received a lot of attention is the design of stochastic RA schemes for wireless networks by solving a suitably defined utility maximization problem. Relevant works in this area include [30], [7], [19], [8], [27], [6], and [9]. Using principles of optimization theory, duality, dynamic control and adaptive signal processing, these works develop stochastic schemes that achieve optimality and stabilize the network. Although related, the algorithms developed here are different because: (i) schemes are tailored for a specific CR scenario; (ii) a novel dual-only approach is introduced that does not require any primal iteration; (iii) power and rate are jointly adapted to meet a prescribed bit error rate (BER); (iv) maximum rate and power constraints are introduced. (Incorporation of power constraints to the utility maximization problem had been mentioned in [9] and [6].); and (v) schemes are developed to account explicitly for Q-CSI. The latter is challenging because rate and power functions under Q-CSI are non-Lipschitz continuous which renders convergence non-trivial to establish. Last but not least, the aforementioned works mainly focus on stochastic algorithms while the present approach is equally applicable to non-stochastic designs, along the lines of e.g., [20] and [23]. The non-stochastic schemes give rise to algorithms where the dual prices are computed off-line and *stationary* resource allocation schemes that only depend on the channel state.

The rest of the paper is organized as follows. After introducing preliminaries on the setup in Section II, we formulate the optimization problem and develop the optimal resource allocation as a function of the channel conditions and the dual prices in Section III. Computation of the optimal dual prices when the channel statistics are known is addressed in Section IV.A, whereas a convergent stochastic algorithm for the case where channel statistics are unknown is the subject of Section IV.B. Partially distributed implementations and channel quantization that allow for reduced feedback overhead are discussed in Section V. Numerical results and comparisons corroborating the analytical claims are presented in Section VI. Concluding remarks in Section VII wrap up this paper.

Notation: Lower- and upper-case boldface fonts are used to denote (column) vectors and matrices, respectively; $(\cdot)^T$ denotes transpose and $(\cdot)^\dagger$ conjugate; $\mathbf{X} \geq \mathbf{0}$ means all entries of $\mathbf{X}$ are nonnegative; $F_{\mathbf{X}}(\mathbf{X})$ denotes the joint cumulative distribution function (CDF) of matrix $\mathbf{X}$; likewise, $F_x(x)$ denotes the CDF of a scalar x; $E_{\mathbf{X}}[\cdot]$ stands for the expectation operator over $\mathbf{X}$; $\lfloor\cdot\rfloor$ ($\lceil\cdot\rceil$) denotes the floor (ceiling) operation; $\mathbf{I}_{\{\cdot\}}$ is short for the indicator function; i.e., $\mathbf{I}_{\{x\}} = 1$ if x is true and zero otherwise; $[x]^+ := \min(x, 0)$; $\beta_n \downarrow 0$ denotes a sequence converging to 0; and $\mathcal{O}(\cdot)$ and $o(\cdot)$ stand for the Landau's big "O" and little "o" orders.

II. Modeling Preliminaries

Consider an OFDMA air interface between an access point (AP) equipped with a central scheduler and J wireless users. Users $j = 1, \ldots, J_p$ are primary spectrum holders and users $j = J_p + 1, \ldots, J$ are secondary ones as in the spectrum overlay paradigm. The overall bandwidth B is divided into K orthogonal narrow-band subcarriers, each with bandwidth B/K small enough to ensure that the fading channel on it is flat, i.e., non-selective. The wireless link between the AP and user j at subcarrier $k = 1, \ldots, K$ is characterized by its random square magnitude $h_{j,k}$, which is assumed normalized by the receiver noise variance. The overall $JK \times 1$ gain vector $\mathbf{h} := \{h_{j,k},\ j = 1, \ldots, J,\ k = 1, \ldots, K\}$ is stationary and ergodic with joint CDF $F(\mathbf{h})$.

Per subcarrier $k = 1, \ldots, K$, we introduce a non-negative time-sharing vector $\boldsymbol{\tau}_k(\mathbf{h}) := \{\tau_{j,k}(\mathbf{h}),\ j = 1, \ldots, J\}$, where entries $\tau_{j,k}(\mathbf{h})$ depend on the channel realization $\mathbf{h}$ and obey the constraint $\sum_{j=1}^{J} \tau_{j,k}(\mathbf{h}) \leq 1$. The fraction $\tau_{j,k}(\mathbf{h})$ represents the percentage (of time) user j gains access to subcarrier k per realization $\mathbf{h}$ over the channel coherence interval; and the constraint $\sum_{j=1}^{J} \tau_{j,k}(\mathbf{h}) \leq 1$ ensures that when several users are scheduled over the same subcarrier, the

duration per channel coherence interval can be split among users (time-shared) so that the access over time[2] remains orthogonal. If scheduled, i.e., if $\tau_{j,k}(\mathbf{h}) > 0$, user j transmits on subcarrier k with rate $r_{j,k}(\mathbf{h})$ and power $p_{j,k}(\mathbf{h})$.

The AP acquires with a sufficient number of training symbols the CSI vector $\mathbf{h}$ based on which it optimizes resource triplets $\{\tau_{j,k}(\mathbf{h}),\ r_{j,k}(\mathbf{h}), p_{j,k}(\mathbf{h})\ \forall\ j,k\}$, and feeds back the optimal schedule to the users using a finite number of bits. This limited-rate feedback enables channel-adaptive operation based on a finite number of possible transmit-configurations. Tailored to such a set-up, let $\mathcal{S}$ denote a set containing a finite number of adaptive modulation, coding, and power (AMCP)[3] combinations (modes). Specifically, let the mth AMCP mode for the jth user on subcarrier k consist of: (i) a chosen modulation (e.g., 16-QAM) and a channel code (e.g., a convolutional code with rate 1/2) with overall rate $r_{j,k,m}$; and (ii) a discrete power level $p_{j,k,m}$. Therefore, the set of AMCP modes is defined as $\mathcal{S} := \{(r_{j,k,m}, p_{j,k,m})\ | j = 1,\ldots,J,\ m = 1,\ldots,M_{j,k},\ k = 1,\ldots,K\}$ where $m = 1,\ldots,M_{j,k}$ indicates that the AMCP modes can be different for each user-subcarrier pair. We will find it convenient to extend the definition of $\mathcal{S}$ and include a fictitious user $j = 0$ with $r_{0,m,k} = 0, p_{0,m,k} = 0$ and $M_{0,k} = 1\ \forall k$, representing an inactive transmitter. This extended definition of $\mathcal{S}$ will allow us to deal with the case where no user transmits on subcarrier k. Throughout Sections III and IV it will be assumed that the codebook $\mathcal{S}$ of rates and powers is prescribed, while in Section V an algorithm will be presented to optimize its construction.

The finite cardinality of $\mathcal{S}$ does not limit users to utilize transmit-rates and powers constrained to a specific AMCP mode (i.e., $r_{j,k,m}$ and $p_{j,k,m}$) since they can naturally support (under the prescribed BER) transmit-rates expressed as linear combinations of these AMCP modes by time-sharing their usage per subcarrier k. Specifically, using the mode m over $\zeta_{j,k,m}$ percentage of the $\tau_{j,k}$ time fraction, and letting $\tau_{j,k,m} := \zeta_{j,k,m}\tau_{j,k}$, user j can support rate

$$r_{j,k}(\mathbf{h}) = \sum_{m=1}^{M_{j,k}} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m} \tag{1}$$

where clearly $\sum_{j=1}^{J}\sum_{m=1}^{M_{j,k}} \tau_{j,k,m} \in [0,1]$; and now the time-allocation vector is defined as $\boldsymbol{\tau}(\mathbf{h}) := \{\tau_{j,k,m}(\mathbf{h}),\ j = 1,\ldots,J,\ m = 1,\ldots,M_{j,k},\ k = 1,\ldots,K\}$. Through time-sharing, any linear combination of rates $r_{j,k,m}$ as in (1) gives rise to the same linear combination of corresponding powers $p_{j,k,m}$; hence,

$$p_{j,k}(\mathbf{h}) = \sum_{m=1}^{M_{j,k}} \tau_{j,k,m}(\mathbf{h}) p_{j,k,m}. \tag{2}$$

Based on the instantaneous rate and power resources in (1) and (2), the average rate and average power of user $j = 1,\ldots,J$ are expressed, respectively, as

$$\bar{r}_j := E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m=1}^{M_{j,k}} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m}\right] \tag{3}$$

$$\bar{p}_j := E_{\mathbf{h}}\left[\sum_{k=1}^{K}\sum_{m=1}^{M_{j,k}} \tau_{j,k,m}(\mathbf{h}) p_{j,k,m}\right]. \tag{4}$$

III. Channel-adaptive resource allocation

The optimal resource allocation will be obtained in this section as the solution of a constrained optimization problem. The *objective* of this problem will be based on concave and increasing so called *utility functions* $U_j(\bar{r}_j)$, that are commonly used in resource allocation tasks (not only restricted to communication systems), and account for the "social" utility (reward) that a specific resource gives rise to. On the other hand, to guarantee QoS, reliability of the wireless links will be maintained under a *maximum* allowable BER $\check{\epsilon}_j$, which in principle can be different per user j. Furthermore, to respect primary/secondary CR hierarchies, a *minimum* average rate $\check{r}_j$ will be enforced for primary user transmissions indexed by $j = 1,\ldots,J_p$; while to prevent secondary users from "abusing" the spectrum, *maximum* average rates $\check{r}_j$ will be imposed for these users too indexed by $j = J_p+1,\ldots,J$. Finally, *maximum* individual average power constraints $\check{p}_j$ will be present for both primary and secondary users.

Then the optimal allocation maximizes the total utility subject to (s.to) average rate and power constraints:

$$\begin{cases} \max\limits_{\boldsymbol{\tau}(\mathbf{h}),\mathbf{r}(\mathbf{h}),\mathbf{p}(\mathbf{h})} & \sum_{j=1}^{J} U_j(\bar{r}_j) \\ \text{s.to} & C1.\ \ \check{r}_j \geq \check{r}_j,\ j = 1,\ldots,J_p \\ & C2.\ \ \bar{r}_j \leq \check{r}_j,\ j = J_p+1,\ldots,J \\ & C3.\ \ \bar{p}_j \leq \check{p}_j,\ j = 1,\ldots,J \\ & C4.\ \ \boldsymbol{\tau}(\mathbf{h}) \in \mathcal{F}_\tau \\ & C5.\ \ \boldsymbol{\tau}(\mathbf{h}), \mathbf{r}(\mathbf{h}), \mathbf{p}(\mathbf{h}) \in \mathcal{F}_\varepsilon \end{cases} \tag{5}$$

where constraints $C1$ and $C2$ enforce the primary-secondary CR hierarchies; constraints $C3$ ensure adherence to the power budget of individual users; constraints $C4$ dictate the user allocation to be feasible, i.e., $\mathcal{F}_\tau$ represents the time policies so that the total usage of each subcarrier cannot exceed one; and the last constraint ensures reliability of the transmissions by satisfying a minimum BER, i.e., $\mathcal{F}_\varepsilon$ represents the allocation policies that satisfy the BER requirement. Note that the last constraint has to be imposed since users rely on AMCP modes instead of capacity-achieving coded transmissions.

To solve (5), we will re-formulate the original optimization problem considering the following issues:

- Constraint $C5$ can be easily satisfied provided that per channel realization $\mathbf{h}$ only the AMCP modes meeting the required BER are considered in the optimization task. With $\epsilon_{j,k,m}(p_{j,k,m}, r_{j,k,m}|h_{j,k})$ denoting the instantaneous BER expressed as a convex function of the channel gain, the transmit-power and rate per channel

[2]For existing OFDMA systems, typical bounds on the coherence and symbol intervals are 5-100 ms and 5-500 μs, respectively. This means that during a coherence interval several hundreds of symbols are transmitted; hence, those symbols can be assigned to different users.

[3]The utilization of discrete AMC modes for wireless communication systems was introduced by [11]. Recently, systems where the transmit-power is also adapted using a finite set of discrete power levels have been investigated by, e.g., [21] for single-user systems and [34] for multi-user systems.

realization $\mathbf{h}$, we can define the set of modes per terminal j and subcarrier k

$$\mathcal{M}_{j,k}(h_{j,k}) := \{m : \epsilon_{j,k,m}(p_{j,k,m}, r_{j,k,m}|h_{j,k}) \leq \check{\epsilon}_j\} \tag{6}$$

that satisfies its BER requirement. The triplet of subscripts in $\epsilon_{j,k,m}$ signify that different modulation and coding schemes are allowed for each user, mode, and subcarrier[4].

- Feasibility of the time allocation policy in $C4$ can be easily described by the set of constraints

$$\tau_{j,k,m}(\mathbf{h}) \geq 0 \ \forall \ \mathbf{h}, j, k, m \quad \text{and}$$
$$\sum_{j=1}^{J} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) \leq 1; \ \forall \ \mathbf{h}, k. \tag{7}$$

  Different from $C1 - C3$, these constraints have to be satisfied for each and every realization $\mathbf{h}$.
- All the optimization variables in (5) can be expressed as linear combinations of $\boldsymbol{\tau}(\mathbf{h})$ and the mode pairs $(r_{j,k,m}, p_{j,k,m}) \in \mathcal{S}$ [cf. (1)-(4)]. Since all the elements of $\mathcal{S}$ are known a priori, the only optimization variable is $\boldsymbol{\tau}(\mathbf{h})$. Once $\boldsymbol{\tau}^*(\mathbf{h})$ is found, $\mathbf{r}^*(\mathbf{h}) = \mathbf{r}(\boldsymbol{\tau}^*(\mathbf{h}))$ and $\mathbf{p}^*(\mathbf{h}) = \mathbf{p}(\boldsymbol{\tau}^*(\mathbf{h}))$ can be obtained.

Based on these considerations and after substituting (3) and (4) into (5), the optimal management of resources can be determined as the solution of the following constrained optimization problem:

$$\begin{cases} \max\limits_{\boldsymbol{\tau}(\mathbf{h})} \ \sum_{j=1}^{J} U_j \left(E_{\mathbf{h}} \left[\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m} \right] \right) \\ \text{s.to} \quad C1. \ \ E_{\mathbf{h}} \left[\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m} \right] \geq \check{r}_j, \\ \qquad\qquad j = 1, \ldots, J_p \\ \qquad C2. \ \ E_{\mathbf{h}} \left[\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m} \right] \leq \check{r}_j, \\ \qquad\qquad j = J_p + 1, \ldots, J \\ \qquad C3. \ \ E_{\mathbf{h}} \left[\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) p_{j,k,m} \right] \leq \check{p}_j, \\ \qquad\qquad \forall j \\ \qquad C4.1. \ \ \tau_{j,k,m}(\mathbf{h}) \geq 0; \ \forall \mathbf{h}, \ \forall j, k, m \\ \qquad C4.2. \ \ \sum_{j=1}^{J} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) \leq 1; \ \forall \mathbf{h}, \ \forall k. \end{cases} \tag{8}$$

The problem formulated as in (8) is convex and can be efficiently solved using a Lagrange multiplier based primal-dual approach [5, Sec. 5.1]. Note that in principle, $C1$ and $C2$ (which enforce the hierarchy between primary and secondary users) entail a serious threat to convexity. This is because the same function is constrained in opposite directions – the average rate is lower-bounded in $C1$ and upper-bounded in $C2$. As a result, the original problem can be convex only if such a function is linear w.r.t. the optimization variables. Due to the operating conditions considered in this paper, namely orthogonal access and transmissions based on AMCP modes, this is indeed the case [c.f. (3) and (4)], and convexity of (8) can thus be ensured.

Note that average rate constraints in (5) and (8) guarantee that the *average* information rate of a primary user remains above a given requirement. However, there is no guarantee about the *instantaneous* transmit-rate. In other words, the formulation in (8) implicitly assumes that users are equipped with infinitely backlogged queues, where the information to be sent is stored. As a result, only non-real time traffic without instantaneous short-term delay requirements can be accommodated. Likewise, the solvers of (8) developed in the ensuing sections presume that the problem at hand is feasible too. If the minimum rate requirements of primary users are too high, and their power budgets too small, the optimization in (8) could be infeasible. This however is readily detectable, since the Lagrange multipliers associated with some of the infeasible users would grow unbounded. Clearly, in such a case the only option to stabilize the system resorts to dropping some primary users. Unfortunately, the problem of selecting the optimum users to drop (a.k.a. admission control) is often NP-hard and goes beyond the scope of this work.

Remark 1: Sum-utility maximization has been employed by scheduling, MAC layer, and networking algorithms; see e.g., [22], [16], [9], and references therein. A special case of (5) and (8) occurs when the utility function takes the linear form $U_j(\bar{r}_j) := w_j \bar{r}_j$, with $w_j \geq 0$ representing a *rate-reward weight* whose value can be tuned to effect fairness and priority. If $U_j(\bar{r}_j) := w_j \bar{r}_j$ and $C1$ and $C2$ are not present (i.e., $\check{r}_j = 0$ if $j \leq J_p$ and $\check{r}_j = \infty$ if $j > J_p$), then (5) reduces to the classical weighted sum-rate maximization problem encountered in information-theoretic studies; see e.g., [20]. Furthermore, it is worth emphasizing that: (i) the results we will derive can be applied to the weighted-sum-rate maximization problem; and (ii) we will find links between the two problems and provide intuition behind the weight vector $\boldsymbol{w} := [w_1, \ldots, w_J]^T$.

Remark 2: OFDMA entails transmit-waveforms $\{g_j(t)\}_{j=1}^{J}$ corresponding to the complex exponential basis $\{e^{j2\pi \frac{(k-1)t}{KT_s}}\}_{k=1}^{K}$. Recent works have shown how the complexity of sensing and waveform design in CR can be reduced if users represent $\{g_j(t)\}_{j=1}^{J}$ using a basis $\{\psi_k(t)\}_{k=1}^{K}$ tailored to the intended propagation channel [31]. Interestingly, the formulation of (5) can be also applied to bases other than the exponential. Differences arise only in interpreting the physical meaning of the variables involved. Specifically, $\sqrt{h_{j,k}}$ has to be interpreted as the projection of the jth user's channel $h_j(t)$ over the kth member of the conjugate basis (†), i.e., $\sqrt{h_{j,k}} := \int_{nKT_s}^{(n+1)KT_s} h_j(t)\psi_k^\dagger(t)dt$ with n indexing symbols; and $\sqrt{p_{j,k}(\mathbf{h})}$ must be understood as the coefficient that the jth user utilizes to weigh the kth element of the basis, i.e., $g_j(t) = \sum_{k=1}^{K} \sqrt{p_{j,k}(\mathbf{h})}\psi_k(t)$. This in turn implies that once a specific basis is selected, solving (5) yields not only the optimum resource allocation parameters, but also the optimally designed transmit-waveforms.

[4]By selecting the AMCP modes as in (6), the BER requirement is satisfied per channel realization. Alternatively, one could constrain the average BER across all channel realizations. Average BER constraints however, create non-linear coupling among $r_{j,k,m}$, $p_{j,k,m}$ and $\tau_{j,k,m}$ for a given user j, rendering the problem harder to solve. On the other hand, since the inequality in (6) is a stricter constraint, the total utility could be slightly smaller. However, for related optimization problems it has been established that this degradation is practically negligible [25].

A. Characterizing the optimum channel-adaptive resource management

Let λ_{r_j} and λ_{p_j} denote the Lagrange multipliers associated with average rate and power constraints of the primary ($j = 1, \ldots, J_p$) and secondary ($j = J_p + 1, \ldots, J$) users. Ignoring temporarily the instantaneous constraints $C4$, the Lagrangian is a function of $\boldsymbol{\tau}$ and $\boldsymbol{\lambda} := [\lambda_{r_1}, \lambda_{p_1}, \ldots, \lambda_{r_J}, \lambda_{p_J}]^T$ given by

$$L(\boldsymbol{\lambda}, \boldsymbol{\tau}) := \sum_{j=1}^{J} U_j(\bar{r}_j(\boldsymbol{\tau})) + \sum_{j=1}^{J_p} \lambda_{r_j}(\bar{r}_j(\boldsymbol{\tau}) - \check{r}_j) - \sum_{j=J_p+1}^{J} \lambda_{r_j}(\bar{r}_j(\boldsymbol{\tau}) - \check{r}_j) - \sum_{j=1}^{J} \lambda_{p_j}(\bar{p}_j(\boldsymbol{\tau}) - \check{p}_j). \quad (9)$$

The Lagrange dual function is

$$D(\boldsymbol{\lambda}) := \max_{\boldsymbol{\tau} \in \boldsymbol{\mathcal{F}}_{\boldsymbol{\tau}}} L(\boldsymbol{\lambda}, \boldsymbol{\tau}) \quad (10)$$

where $C4.1$ and $C4.2$ representing the feasible policies in $\mathcal{F}_\tau$ are imposed explicitly since they were not considered in the Lagrangian. Finally, with $\boldsymbol{\lambda} \geq \mathbf{0}$ denoting that all entries of $\boldsymbol{\lambda}$ are non-negative, the dual problem of (8) is

$$\min_{\boldsymbol{\lambda} \geq \mathbf{0}} D(\boldsymbol{\lambda}). \quad (11)$$

Since our problem is convex and strict feasibility is assumed, solving the unconstrained problem in (11) amounts to solving the original constrained problem in (8). But to solve (11), we will need first to solve the maximization in (10). Given $\boldsymbol{\lambda}$, the optimum time allocation $\tau^*_{j,k,m}(\boldsymbol{\lambda}, \mathbf{h})$ depends on both the current channel realization and the value of the multipliers. Upon substituting $\boldsymbol{\tau}^*(\boldsymbol{\lambda}, \mathbf{h})$ $\forall \mathbf{h}$ into (3) and (4) we can find the optimum values of the average rate and power $\bar{r}^*_j(\boldsymbol{\lambda})$ and $\bar{p}^*_j(\boldsymbol{\lambda})$, which generally depend on $\boldsymbol{\lambda}$. Moreover, for future use let us introduce user-specific weights defined through the derivative U'_j as

$$w_j(\boldsymbol{\lambda}) := U'_j(\bar{r}^*_j(\boldsymbol{\lambda})). \quad (12)$$

Clearly, if the utility function is linear, then w_j is a constant not dependent on $\boldsymbol{\lambda}$. The reason behind introducing (12) will be apparent in Section IV-A when the algorithm to find the optimum $\boldsymbol{\lambda}^*$ will be presented and $\boldsymbol{w}$ will be recast as an auxiliary dual variable.

To express the solution of (10), it is useful to introduce what we term link quality indicators

$$\begin{aligned} \varphi_{j,k,m}(\boldsymbol{\lambda}, \mathbf{h}) &:= \varphi_{j,k,m}(\lambda_j, w_j(\boldsymbol{\lambda}), h_{j,k}) \\ &:= (w_j(\boldsymbol{\lambda}) + \lambda_{r_j}) r_{j,k,m} - \lambda_{p_j} p_{j,k,m}, \\ &\quad \forall m \in \mathcal{M}_{j,k}(h_{j,k}),\ j = 1, \ldots, J_p \quad (13) \\ \varphi_{j,k,m}(\boldsymbol{\lambda}, \mathbf{h}) &:= \varphi_{j,k,m}(\lambda_j, w_j(\boldsymbol{\lambda}), h_{j,k}) \\ &:= (w_j(\boldsymbol{\lambda}) - \lambda_{r_j}) r_{j,k,m} - \lambda_{p_j} p_{j,k,m}, \\ &\quad \forall m \in \mathcal{M}_{j,k}(h_{j,k}),\ j = J_p + 1, \ldots, J; \quad (14) \end{aligned}$$

where by construction $\varphi_{0,m,k} = 0$, and using (12) $\varphi_{j,k,m}$ can be expressed also as a function of U'_j. Per subcarrier k, we determine for each user $j = 1, \ldots, J$ the "most-efficient" mode in the sense that

$$m^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) := \arg \max_{m \in \mathcal{M}_{j,k}(h_{j,k})} \varphi_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}); \quad (15)$$

and select the "most-efficient" user as the one with index

$$j^*_k(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) := \arg \max_j \varphi_{j,k,m^*_{j,k}}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}). \quad (16)$$

Per sub-carrier k, the optimal schedule of time-sharing fractions turns out to be (cf. Appendix A)

$$\tau^*_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) = \begin{cases} 1, & \text{if } \ j = j^*_k \text{ and } m = m^*_{j^*_k,k} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

i.e., the "most-efficient" user is the only user gaining access to the subcarrier k. For this reason, user j^*_k will be termed "winner user" of subcarrier k.

Appendix A contains the derivation of (17) and shows that: (i) the allocation in (17) *always* maximizes the Lagrangian in (10); and (ii) if more than one user attain the maximum, the policy of allowing only one of them accessing each subcarrier is still optimum. Specifically, if a tie occurs, the user selected for transmission can be randomly chosen among the multiple winners so that the QoS constraints are met with equality[5].

Substituting the optimal time allocation (17) into (1) and (2), it is possible to express the optimum transmit-rate and power per user and subcarrier as

$$r^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) := \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau^*_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) r_{j,k,m} \quad (18)$$

$$p^*_{j,k}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) := \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau^*_{j,k,m}(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h}) p_{j,k,m} \quad (19)$$

where indeed for any given triplet $(\boldsymbol{\lambda}, \boldsymbol{w}(\boldsymbol{\lambda}), \mathbf{h})$, the optimal schedule lets terminal j^*_k exclusively transmit with its most efficient rate-power pair while having all other terminals $j \neq j^*_k$ defer on subcarrier k.

Once the primal solution of (10) has been found, the dual problem (11) can be solved to obtain the optimal multipliers $\boldsymbol{\lambda}^*$. The complementary slackness condition [5, Sec. 5.5.2], implies that the optimal multipliers in (11) must satisfy

$$\bar{r}_j(\boldsymbol{\lambda}^*) = \check{r}_j \quad \text{or} \quad \lambda^*_{r_j} = 0,\ j = 1, \ldots, J \quad (20)$$

$$\bar{p}_j(\boldsymbol{\lambda}^*) = \check{p}_j \quad \text{or} \quad \lambda^*_{p_j} = 0,\ j = 1, \ldots, J, \quad (21)$$

where having any $\lambda^*_{r_j}$ or $\lambda^*_{p_j}$ equal to zero means that the corresponding constraint is inactive; i.e., it is naturally satisfied without being explicitly imposed. Equally interesting, if the problem is infeasible the value of the corresponding multipliers will grow to infinity. This lends itself naturally to an admission control policy, i.e., to a criterion for dropping users or QoS requirements that render the problem infeasible [35].

[5] A more detailed explanation of this "tie resolution" can be found in Appendix A; however, it is worth mentioning that if the AMCP modes are linearly independent, the probability of this event vanishes as J, K or $M_{j,k}$ grow large.

The non-linear system of (13)-(21) completely characterizes the optimum allocation parameters. At the optimum, these equations can be viewed as the Karush-Kuhn-Tucker (KKT) conditions of (8). Even though with $\boldsymbol{\lambda}$ and $\boldsymbol{w}(\boldsymbol{\lambda})$ fixed the remaining variables can be analytically found [cf. (13)-(17)], there is no analytical solution for the system when the dual prices $\boldsymbol{\lambda}$ and $\boldsymbol{w}(\boldsymbol{\lambda})$ are also considered as variables. As a result, one has to resort to numerical search algorithms to find the jointly optimal solution. In the next section, we develop two such convergent algorithms to find the optimum $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}(\boldsymbol{\lambda}^*)$. Recall that once the optimum dual prices $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}(\boldsymbol{\lambda}^*)$ are found, (13)-(17) yield the optimum channel-adaptive resource allocation parameters; i.e., the optimum management of resources as a function of $\mathbf{h}$. A remark is now due on the structure of the link quality indicator in (13)-(14), which relates the winner-takes-all strategy in (17) with the optimal solution of other resource allocation problems.

Remark 3: Regarding λ_{r_j} and λ_{p_j} as prices of the rate and power and $w_j(\boldsymbol{\lambda}) = U_j'(\bar{r}_j(\boldsymbol{\lambda}))$ as a rate-weight representing the marginal utility per transmitted bit[6], the link quality indicators in (13) and (14) determine the net rate reward (rate reward minus power cost) of the (j,m)th mode on subcarrier k. This means that users with large $w_j(\boldsymbol{\lambda}) = U_j'(\bar{r}_j(\boldsymbol{\lambda}))$ are promoted for selection since they contribute a lot to the increase of the total utility. In a secondary market CR set-up, to satisfy the *individual* QoS per user, the marginal utility of the primary users $j \leq J_p$ is promoted through the additive multiplier $\lambda_{r_j} > 0$; whereas these positive multipliers are subtracted from the marginal utility to prevent abusive spectrum access by secondary users $j > J_p$. (Likewise, $\lambda_{p_j} > 0$ can be always viewed as a penalty or cost.) Using such indicators, the optimal allocation maximizes the *average* utility so that *per channel realization* $\mathbf{h}$ each subcarrier k is uniquely assigned to the winning user-mode pair $(j_k^*, m_{j_k^*,k}^*)$ with the highest net rate reward. Further, it must be emphasized that neither the structure of the channel link quality indicators in (13) and (14) nor the values (prices) of w_j, λ_{r_j}, and λ_{p_j} have been imposed *a fortiori*. Instead, they emerge from the optimal solution of the problem in (8). This in turn implies that the optimal management of resources depends only on the current channel realization $\mathbf{h}$ and on the dual user-specific prices (correspondingly rewards) $\boldsymbol{\lambda}$ ($\boldsymbol{w}$).

The winner-takes-all strategy has been shown to be optimal for other problems that also deal with orthogonal sharing of resources among users. For example, in the context of multi-user wireless channels, a related scheduling that maximizes ergodic capacity subject to average power constraints can be found in [20]. Similarly, from a network utility maximization perspective, (17) can be viewed as an enhancement of the max-weight scheduling; see e.g., [19], [30]. In fact, some of these works use instantaneous values of the queue lengths instead of dual price values; see e.g., [9] and references therein.

[6]Using the approximation $U_j'(\bar{r}_j) \doteq \Delta U_j / \Delta r_j$, with $\Delta r_j = 1$, the weight reduces to $w_j(\boldsymbol{\lambda}) = \Delta U_j$.

IV. Finding the optimal dual prices

In this section, we present two algorithms for finding the optimal dual prices $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}(\boldsymbol{\lambda}^*)$. The first relies on a subgradient iteration which exploits the knowledge of the channel CDF $F(\mathbf{h})$. The second relies on an adaptive LMS-like iteration and does not require knowledge of the channel statistics. For both algorithms, convergence is analyzed and differences are identified.

A. Channel statistics known: off-line calculation

Since the problem in (8) is convex and maximization of the associated Lagrangian can be obtained uniquely, $\boldsymbol{\lambda}^*$ can be found by iterating over the dual function which is always convex and its global optimum can be found using (sub)gradient iterations [4, Chap. 6]. However, since (13), (14), (20), and (21) involve expectations over the channel gains, knowledge of the channel CDF is required. Furthermore, although uniquely defined by (13)-(17), finding the optimal primal variable $\boldsymbol{\tau}^*(\boldsymbol{\lambda}, \mathbf{h})$ in each dual iteration is numerically non-trivial. This is because finding the optimum time allocation $\boldsymbol{\tau}^*(\boldsymbol{\lambda}, \mathbf{h})$ on the one hand, requires the optimum weight vector $\boldsymbol{w}(\boldsymbol{\lambda})$ [cf. (17)]; while on the other hand, finding $\boldsymbol{w}(\boldsymbol{\lambda})$ one needs $\bar{\mathbf{r}}^*(\boldsymbol{\lambda})$ and therefore $\boldsymbol{\tau}^*(\boldsymbol{\lambda}, \mathbf{h})$ $\forall \mathbf{h}$ [cf. (12) and (3)]. Although this "chicken-egg" dilemma can be resolved through a nested iterative search (recall that the solution is uniquely defined), the computational burden is considerably high. This burden can be lightened by defining $\boldsymbol{w}(\boldsymbol{\lambda})$ as a separate auxiliary variable $\boldsymbol{w}$ that does not depend on $\boldsymbol{\lambda}$. To this end, we introduce in Appendix B an equivalent formulation of (8) where $\boldsymbol{w}$ is treated as a dual variable and its determination can thus be decoupled from that of $\boldsymbol{\tau}^*(\boldsymbol{\lambda}, \mathbf{h})$.

Once $\boldsymbol{w}$ is treated as a dual variable, the new dual function depends both on $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ [cf. Appendix B]. To implement the resultant numerical search over dual prices, let i denote the iteration index and $\beta^{(i)}$ a small decreasing stepsize. With $U_j'^{-1}$ representing the inverse function of U_j', define also the rate-weight function as $\tilde{r}_j(w_j) := U_j'^{-1}(w_j)$. Intuitively speaking, $\tilde{r}_j(w_j)$ represents the average rate for which the weight w_j is optimum [cf. (12)]. Note that for $U_j(\cdot)$ monotonically increasing and strictly convex, $U_j'^{-1}(\cdot)$ (thus $\tilde{r}_j(\cdot)$) is positive and monotonically decreasing. Then, assuming that the channel CDF is known, the optimal off-line solution for $\boldsymbol{w}$ and $\boldsymbol{\lambda}$ can be found through the iterations

$$\lambda_{r_j}^{(i+1)} = \left[\lambda_{r_j}^{(i)} - \beta^{(i)}\left(E_{\mathbf{h}}\left[\sum_{k=1}^{K} r_{j,k}^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{w}^{(i)}, \mathbf{h})\right] - \check{r}_j\right)\right]^+, \quad j = 1, \ldots, J_p \tag{22}$$

$$\lambda_{r_j}^{(i+1)} = \left[\lambda_{r_j}^{(i)} - \beta^{(i)}\left(\check{r}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} r_{j,k}^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{w}^{(i)}, \mathbf{h})\right]\right)\right]^+, \quad j = J_p + 1, \ldots, J \tag{23}$$

$$\lambda_{p_j}^{(i+1)} = \left[\lambda_{p_j}^{(i)} - \beta^{(i)}\left(\check{p}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} p_{j,k}^*(\boldsymbol{\lambda}^{(i)}, \boldsymbol{w}^{(i)}, \mathbf{h})\right]\right)\right]^+, \quad j = 1, \ldots, J \tag{24}$$

$$w_j^{(i+1)}=\left[w_j^{(i)}-\beta^{(i)}\left(\tilde{r}_j(\boldsymbol{w}^{(i)})-E_{\mathbf{h}}\left[\sum_{k=1}^{K} r_{j,k}^*(\boldsymbol{\lambda}^{(i)},\boldsymbol{w}^{(i)},\mathbf{h})\right]\right)\right]^+, \quad j=1,\ldots,J. \tag{25}$$

If the utility is linear (not strictly convex), then w_j is constant and the update in (25) is not required. Notice that each iteration involves expectations over the channel CDF and requires first computing the optimal allocation for all $\mathbf{h}$ after substituting $\boldsymbol{\lambda}^{(i)}$ and $\boldsymbol{w}^{(i)}$ (available from the previous iteration i) into (18) and (19). Using convergence results from convex optimization [4, Chap. 6] we have the following claims:

Theorem 1: *If the convex problem (8) is strictly feasible, the updates (22)-(25) represent subgradient iterations whose fast (linear) convergence to the optimal $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$ as i increases and $\beta^{(i)}$ decreases is guaranteed from any initial positive value, $\boldsymbol{\lambda}^{(0)} \geq \mathbf{0}$, $\boldsymbol{w}^{(0)} \geq \mathbf{0}$. After the optimum values $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$ are found, the optimal resource allocation per channel realization is in turn provided by $\boldsymbol{\tau}^*(\mathbf{h}) = \boldsymbol{\tau}^*(\boldsymbol{\lambda}^*, \boldsymbol{w}^*, \mathbf{h})$.*

Proof: See Appendix B. ■

To interpret Theorem 1, recall first that due to the convexity of (8) the optimal solution can be found by solving (11). Therefore, since (22)-(25) correspond to subgradient iterations of the unconstrained convex problem in (11), their convergence to the optimum is guaranteed (the same can be argued for the augmented problem where $\boldsymbol{w}$ is recast as a dual variable). The iterations (22)-(25) are typically run *off-line* (i.e., before the communication starts) during the initialization phase of the system, or, whenever the CDF of the vector channel changes. If necessary, the standard subgradient (first-order) iterations in (22)-(25) can be replaced by modified versions whose speed of convergence is even faster (see [4, pp. 624-629] for details). However, we advocate the standard subgradient iterations since, besides being convergent, they are simple to implement. This simplicity will also facilitate the development of *stochastic* (Sec. IV-B) and/or partially *distributed* (Sec. V) on-line implementations of the original *off-line* iterations.

It is worth stressing that treating $\boldsymbol{w}$ as a dual variable, not only mitigates the "chicken-egg" problem mentioned at the beginning of this section, but also allows one to optimally find $\boldsymbol{w}^*$ using a subgradient iteration. This unifies all the updates (which now take place in the same (dual) domain and exhibit similar convergence), and renders the allocation policy (13)-(17) dependent only on the realization $\mathbf{h}$, and the dual prices $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$.

There are situations where solving (22)-(25) is not a viable alternative either because $F(\mathbf{h})$ is unknown, or, because the channel statistics do not remain invariant over time; or, the computational burden associated with re-calculating (22)-(25) each time they change can not be afforded. For such scenarios, we go one step further in the next section to develop *fully on-line* solutions that do not require knowledge of the channel CDF and incur negligible computational complexity.

B. Channel statistics unknown: learning the environment on-the-fly

Suppose that the fading channel vector $\mathbf{h}$ remains invariant over a block of OFDMA symbols but can vary from block-to-block (block fading channel model). Let n denote the current block index and $\mathbf{h}[n]$ the fading state during block n, whose duration is dictated by the channel coherence interval. The main difficulty in implementing on-line the optimal channel-adaptive allocation derived in (18)-(19) is that $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ have to be known, which requires an off-line computation based on the channel CDF. To tackle this problem, we will rely on adaptively updated *instantaneous* estimates of $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ that eventually will allow us to bypass the off-line calculation as well as the need to know the channel statistics. To this end, we will replace the expectations in (22)-(25) by standard stochastic approximations. This will allow us to substitute the off-line iteration index i by the instantaneous block (time) index n, and then execute an on-line recursion across blocks to obtain the instantaneous estimates $\hat{\boldsymbol{w}}[n] := [\hat{w}_1[n], \ldots, \hat{w}_J[n]]^T$ and $\hat{\boldsymbol{\lambda}}[n] := [\hat{\lambda}_{r_1}[n], \hat{\lambda}_{p_1}[n], \ldots, \hat{\lambda}_{r_J}[n], \hat{\lambda}_{p_J}[n]]^T$ as in (26)-(29).

Iterates $r_{j,k}^*[n] := r_{j,k}^*(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n])$ and $p_{j,k}^*[n] := p_{j,k}^*(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n])$ in (26)-(29) are computed based on (18) and (19) and represent the current rate and power of the user j on subcarrier k over block n, while stepsize $\beta[n] \in [0,1]$ implements a forgetting effect in the averaging. To find $r_{j,k}^*[n]$ and $p_{j,k}^*[n]$ per block n, the optimum AMCP mode and user for each subcarrier k have to be found by substituting the current $\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n]$ estimates into (13)-(17). Once the allocation parameters of the nth block are obtained, we can use (26)-(29) to update both reward weights $\hat{\boldsymbol{w}}[n+1]$ and Lagrange multipliers $\hat{\boldsymbol{\lambda}}[n+1]$ with negligible (linear in the number of modes, users and subcarriers) computational complexity.

Devoid of the expectation operators, the updates (26)-(29) offer unbiased estimates of the subgradient projections in (22)-(25). Such iterations along with the on-line optimal allocation amount to a stochastic dual (SD) algorithm for solving the utility maximization problem in (5). Per block n, this algorithm performs a weighted sum-rate maximization with adaptive weights provided by $\hat{w}_j[n] + \hat{\lambda}_{r_j}[n]$ and $\hat{\lambda}_{p_j}[n]$ for primary CR users and $\hat{w}_j[n] - \hat{\lambda}_{r_j}[n]$ and $\hat{\lambda}_{p_j}[n]$ for secondary CR users to obtain on-line optimal allocation, whereas the variables $\hat{\boldsymbol{\lambda}}[n+1]$ and $\hat{\boldsymbol{w}}[n+1]$ are updated using *instantaneous* transmit-powers and rates.

Interestingly, without knowing $F(\mathbf{h})$, this simple SD on-line algorithm can learn the channel CDF on-the-fly, and is convergent and asymptotically optimal as the following theorem states.

Theorem 2: *If problem (5) is strictly feasible, then the estimates obtained recursively in (26)-(29) using any initial $\hat{\boldsymbol{\lambda}}[0] \geq \mathbf{0}$ and $\hat{\boldsymbol{w}}[0] \geq \mathbf{0}$, converge in probability to the optimal $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*(\boldsymbol{\lambda}^*)$ of (5), as $n \to \infty$ and $\beta[n] \downarrow 0$.*

Proof: See Appendix C. ■

In order to avoid a premature convergence, the stepsize must satisfy $\sum_{n=0}^{\infty} \beta[n] = \infty$. Equally important, with a small but constant stepsize $\beta[n] = \beta$, the SD algorithm brings $\hat{\boldsymbol{\lambda}}[n]$ to a small neighborhood of $\boldsymbol{\lambda}^*$ (with size $o(\beta)$) in $o(1/\beta)$ iterations, uniformly for any initial state; see Appendix C for detailed explanation. Because this adaptive algorithm converges from arbitrary initializations it exhibits robustness to channel non-stationarities as long as the channel remains stationary for sufficiently long periods, or, the channel CDF varies sufficiently slowly. Compared to the off-line solution,

$$\hat{\lambda}_{r_j}[n+1] = \left[\hat{\lambda}_{r_j}[n] - \beta[n]\left(\sum_{k=1}^{K} r^*_{j,k}(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n]) - \check{r}_j\right)\right]^+, \; j = 1, \ldots, J_p \tag{26}$$

$$\hat{\lambda}_{r_j}[n+1] = \left[\hat{\lambda}_{r_j}[n] - \beta[n]\left(\check{r}_j - \sum_{k=1}^{K} r^*_{j,k}(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n])\right)\right]^+, \; j = J_p+1, \ldots, J \tag{27}$$

$$\hat{\lambda}_{p_j}[n+1] = \left[\hat{\lambda}_{p_j}[n] - \beta[n]\left(\check{p}_j - \sum_{k=1}^{K} p^*_{j,k}(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n])\right)\right]^+, \; j = 1, \ldots, J \tag{28}$$

$$\hat{w}_j[n+1] = \left[\hat{w}_j[n] - \beta[n]\left(\tilde{r}_j\left(\hat{w}_j[n]\right) - \sum_{k=1}^{K} r^*_{j,k}(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h})\right)\right]^+, \; j = 1, \ldots, J. \tag{29}$$

the adaptive SD algorithm enjoys two attractive features: (i) convergence to the optimal average rates without a priori knowledge of the fading CDF, and (ii) ability to adapt the user-mode selection based on the short-term behavior of the channel. For example, if the channel gains for the jth primary terminal are low over consecutive slots, the corresponding dual rate price $\lambda_{r_j}[n]$ will readily rise and the net-reward $\varphi_{j,k,m}$ $\forall m, k$ will correspondingly increase the probability of selecting this terminal even if its channel gains are not as good.

V. On the limited-rate feedback

The optimal resource allocation presented so far can be easily implemented when the scheduler at the access point knows the price vectors $\hat{\boldsymbol{\lambda}}[n]$ and $\hat{\boldsymbol{w}}[n]$, the set of AMCP modes $\mathcal{S}$, and the vector channel realization $\mathbf{h}[n]$. Based on those, in every coherence interval the optimal transmit-configuration per subcarrier ($r^*_{j,k}[n]$, $p^*_{j,k}[n]$, $\forall j, k$) can be computed using (18)-(19) and fed back to the CR user terminals.

Under certain operational conditions however, the feedback required from the access point can be reduced without loss in performance. These conditions are different for TDD and FDD systems, and for this reason will be discussed separately. Because the number of variables that are updated per block index n is higher when the channel CDF's are unknown, in this section we will focus on the algorithms of Section IV-B. The related analysis when $F(\mathbf{h})$ is known is a simplified version of the one presented here, where instead of updating the dual prices each and every n, updates take place only when the channel statistics change.

A. TDD systems

Since for TDD systems uplink and downlink channels are reciprocal, $h_{j,k}[n]$ can be acquired wherever needed by estimating the channel in the reverse link. If the access point knows $\hat{\boldsymbol{\lambda}}[n]$, $\hat{\boldsymbol{w}}[n]$, $\mathcal{S}$ and each terminal j knows its own dual prices
$hatlambda_{r_j}[n]$, $\hat{\lambda}_{p_j}[n]$, $\hat{w}_j[n]$ and AMCP modes $\mathcal{S}_j := \{(r_{j,k,m}, p_{j,k,m}),\ \forall m, k\}$, channel reciprocity can be exploited to reduce the feedback overhead under the following operating conditions:

(oc.1) Both access point and terminals employ: (i) the same initialization for $\hat{\boldsymbol{\lambda}}[0]$ and $\hat{\boldsymbol{w}}[0]$; and (ii) identical stepsize (forgetting factor) $\beta[n]$ $\forall n$.

(oc.2) For each block index n, the receiving access point:
(i) substitutes $\hat{\boldsymbol{\lambda}}[n]$ and $\hat{\boldsymbol{w}}[n]$ into (13)-(17) to find the optimal allocation parameters $\forall j, k$;
(ii) runs the dual updates in (26)-(29) to obtain $\hat{\boldsymbol{\lambda}}[n+1]$ and $\hat{\boldsymbol{w}}[n+1]$ $\forall j, k$; and
(iii) feeds back to the users the message (codeword) $\mathbf{c}_A[n] := [j^*_1[n], \ldots, j^*_K[n]]^T$. Note that $\mathbf{c}_A[n]$ contains *only* the winner user-subcarrier index corresponding to $\tau^*_{j,k}(\hat{\boldsymbol{\lambda}}[n], \hat{\boldsymbol{w}}[n], \mathbf{h}[n])$ $\forall j, k$.

(oc.3) For each block index n, the transmitting terminals:
(i) for each subcarrier k, the winner terminal $j^*_k[n]$ notified by the access point employs its dual prices together with $h_{j,k}[n]$ to find its optimum transmission mode $m^*_k[n]$, while all other terminals set their transmission power and rate on this subcarrier to zero; and
(ii) once every terminal knows its transmit-rate and power $\forall k$, it updates its own $\hat{\lambda}_{r_j}[n+1]$, $\hat{\lambda}_{p_j}[n+1]$, and $\hat{w}_j[n+1]$ using (26)-(29).

It is worth emphasizing that it is possible to implement the novel allocation schemes in such a way because both the optimal resource allocation and the on-line updates depend only on *local* information (available to each user) and the *global* user scheduling decision which is fed back from the access point. Therefore, having each user knowing its own $\hat{\lambda}_{r_j}[n]$, $\hat{\lambda}_{p_j}[n]$, and $\hat{w}_j[n]$ is possible provided the same initialization (i.e., $\hat{\boldsymbol{\lambda}}[0]$, and $\hat{\boldsymbol{w}}[0]$) is used for the dual updates at user terminals and at the access point.

To feed back the optimum user-subcarrier assignment the control feedback link must be capable of carrying $B_A = \lceil \sum_{k=1}^{K} \log_2(J+1) \rceil$ bits per block.

B. FDD systems without channel quantization

For FDD systems the forward and reverse channels are non-reciprocal, and therefore $h_{j,k}[n]$ is not available at the transmitter(s). This means that when users implement step (i) in (oc.3) they do not have sufficient information to find $m^*_{j,k}[n]$. To bypass this, the access point has to incorporate information of the optimum mode in the feedback message, i.e., $\mathbf{c}_B[n] := [j^*_1[n], m^*_1[n], \ldots, j^*_K[n], m^*_K[n]]^T$. Certainly, if the users know $\mathbf{c}_B[n]$, they do not need the value of their local dual prices and therefore (oc.1) and step (ii) in (oc.3) is no longer needed.

Taking into account this augmented feedback message, the rate required for the feedback link between the access point and the user terminals in FDD increases to $B_B = \lceil \sum_{k=1}^{K} \log_2(\sum_{j=0}^{J} M_{j,k}) \rceil$ bits per block.

C. FDD systems with channel quantization

The overall utility of average rates improves as the number of transmit-modes increases. In fact, with carefully designed modes $\mathcal{M}_{j,k}(\mathbf{h})$ and $M_{j,k} \to \infty$, it is possible to even approach the asymptotically optimum water-filling solution in [20]. On the other hand, high values of $M_{j,k}$ require increased feedback rate from the access point to the users. It is clear that CR welcomes adaptation schemes leading to high utility while requiring reduced limited-rate feedback. This prompted us to investigate channel quantization schemes that reduce the required feedback by optimizing the transmit-power and transmit-rate codebooks. Per user j, this calls for optimizing $\mathcal{S}_j$, which so far has been assumed given.

To perform this optimization we will assume that instead of the analog-valued $h_{j,k}$ (P-CSI), the optimization algorithm relies on a quantized value $h_{j,k}^{Q}$ (Q-CSI). This value is found using a channel quantizer and belongs to a set $\mathcal{L}_{j,k}$ with finite cardinality $L_{j,k}$ so that we can write $\mathcal{L}_{j,k} := \{h_{j,k,l}\}_{l=1}^{L_{j,k}}$. Since the set of feasible modes $\mathcal{M}_{j,k}(h_{j,k})$ satisfying the BER constraint in (6) is selected in accordance with $h_{j,k}$, it is necessary to adapt this definition to the quantized set-up. To do so, it is first useful to introduce the function $\epsilon_{j,k,m}^{Q}(p_{j,k,m}, r_{j,k,m}|h_{j,k}^{Q})$ which expresses the BER as a convex function of the power, the rate, and the *quantized* version of the channel. Based on this function, define

$$\mathcal{M}_{j,k}^{Q}(h_{j,k}^{Q}) := \{m : \epsilon_{j,k,m}^{Q}(p_{j,k,m}, r_{j,k,m}|h_{j,k}^{Q} = h_{j,k,l}) \leq \check{\epsilon}_j\} \tag{30}$$

as the set of AMCP modes satisfying the instantaneous BER requirement $\check{\epsilon}_j$.

At this point, we are ready to implement a modified version of the scheme outlined in Section V-A tailored for FDD systems implementing channel quantization.

(oc.4) Both access point and users: (i) use the same $\hat{\boldsymbol{\lambda}}[0]$ and $\hat{\boldsymbol{w}}[0]$; (ii) identical $\beta[n]$ $\forall n$; and (iii) replace $\mathcal{M}_{j,k}(h_{j,k})$ by $\mathcal{M}_{j,k}^{Q}(h_{j,k}^{Q})$ in every step of the resource allocation algorithm.

(oc.5) For each block index n, the receiving access point:
(i) substitutes $\hat{\boldsymbol{\lambda}}[n]$ and $\hat{\boldsymbol{w}}[n]$ into (13)-(17) to find the optimal RA $\forall j,k$;
(ii) runs the dual updates (26)-(29) to obtain $\hat{\boldsymbol{\lambda}}[n+1]$ and $\hat{\boldsymbol{w}}[n+1]$ $\forall j,k$; and
(iii) feeds back to the users the message $\mathbf{c}_C[n] := [j_1^*[n], l_1^*[n], \ldots, j_K^*[n], l_K^*[n],]^T$, where $l_k^*[n] := \{l : h_{j_k^*[n],k}^{Q} = h_{j_k^*[n],k,l}\}$, i.e., $l_k^*[n]$ represents the index of the quantization region the channel gain of the winner user belongs to.

(oc.6) For each block index n, the transmitting user terminals:
(i) the winner terminal $j_k^*[n]$ uses $h_{j,k}^{Q} = h_{j,k,l_k^*[n]}$ (indexed by the access point) plus $\hat{\lambda}_{r_j}[n]$, $\hat{\lambda}_{p_j}[n]$, and $\hat{w}_j[n]$ (locally stored) to find its optimum transmission mode $m_k^*[n]$, while all other users set their transmission power and rate on this subcarrier to zero; and
(ii) once every user terminal knows its transmit-rate and power $\forall k$, it updates its own $\hat{\lambda}_{r_j}[n+1]$, $\hat{\lambda}_{p_j}[n+1]$, and $\hat{w}_j[n+1]$ using (26)-(29).

Using this modifications the rate required for the feedback link from the access point to the users is $B_C = \lceil \sum_{k=1}^{K} \log_2(\sum_{j=0}^{J} L_{j,k}) \rceil$ bits per block with $B_C < B_B$. The number of bits each winner user needs reduces from $\log_2(M_{j,k})$ down to $\log_2(L_{j,k})$, where in general $L_{j,k} \ll M_{j,k}$. Furthermore, since increasing the value of $M_{j,k}$ translates to improved performance but does not increase B_C, for systems that implement (oc.4)-(oc.6) the value of $M_{j,k}$ should be only limited by hardware complexity.

On the other hand, it is easy to see that the total rate-utility is higher for the problem based on P-CSI than for the alternative implementation based on Q-CSI, since the latter is a special case of the former. Interestingly, simulations will confirm that the performance of FDD systems implementing channel quantization is close to that without quantization, even for small values of $L_{j,k}$. This behavior, which has been consistently observed in different resource allocation problems based on Q-CSI (see e.g., [25], [21], [34]), encourages the practical implementation of adaptive schemes based on limited-rate feedback.

Another interesting observation is that after convergence of the dual iterates corresponding to the lth quantization region on the kth subcarrier, the jth user will always utilize the same AMCP mode if selected for transmission on that subcarrier [cf. (30) and (15)]. This means that the jth user will utilize at most $L_{j,k}$ different modes on the kth subcarrier. In other words, for each subcarrier k, the user j will utilize the information contained in the dual prices to select among the $M_{j,k}$ available modes, the $L_{j,k}$ that best fits the specific CR environment. For each user j, this can be viewed as an optimum codebook design, where starting with a codebook $\mathcal{S}_j$ of size $\sum_{k=1} KM_{j,k}$ a new codebook $\mathcal{S}_j^{Q}$ is constructed with reduced size $\sum_{k=1} KL_{j,k}$.

Remark 4: If needed, on top of our quantization design further reduction of the feedback overhead can be effected by exploiting the possible correlation among channel gains: (i) across subcarriers (e.g., by grouping subcarriers and then indexing each group, or, by using more sophisticated schemes that exploit the sparsity of the transmit-signals); and (ii) across time, e.g., by implementing differential quantization techniques that use less (more) bits to index the regions that are closer (further) to the previous one.

Remark 5: Given the channel quantizer, optimization so far was carried over the rate and power codebooks, i.e., the rate and power that a terminal utilizes when its channel belongs to a given region. An alternative could be to jointly optimize over the channel quantizer and the rate/power codebooks. Although the globally optimum solution of this joint design would lead to a larger utility, it requires off-line quantization schemes such as the well-known Lloyd's algorithm (see e.g., [18]), which besides guaranteeing only local convergence it precludes a fully on-line solution. Investigating the design of asymptotically optimum stochastic channel quantizers tailored for CRs is an interesting future research direction, but goes

beyond the scope of this work.

D. On-line overhead

Since the allocation algorithms developed are to be implemented on-line, the involved overhead is a critical issue. Table I summarizes the feedback requirements and the complexity of the three schemes presented in Section V. Note that the feedback rate B is typically a small number for practical CRs. For instance, with one primary and three secondary users, each supporting $M_{j,k} = 30$ AMCP modes and $L_{j,k} = 4$ quantization regions only 2, 7, and 4, feedback bits per subcarrier are required to implement the schemes in Sections V-A, V-B, and V-C, respectively. Regarding computational complexity, the number of operations remains linear in all cases and the load is particulary small at the users' side. Consider the number of operations at the access point in Section V-C (last row and central column of Table I). To implement the three steps in (oc.5), the access point needs the following number of operations. In step (i): KLJ for the quantization; KMJ to find the set of active modes in (6); with M' denoting the number of active modes; $M'KJ$ to find the optimum transmit-mode; and, KJ for finding the optimum user. In step (ii): KJ summations for each dual price; and in step (iii) there are no new calculations. The total number of operations is, $K(L+M+M'+3+1)J < K(3M+4)J$, which is $\mathcal{O}(KMJ)$. The same logic can be followed to count the calculations performed by the users, where the average number of subcarriers a specific terminal utilizes is assumed to be K/J (this is reasonable if users are homogeneous or J is sufficiently large).

Assessment of the computational complexity is different for the off-line algorithm of Section IV-A, specifically because neither the access point nor the users have to implement an on-line algorithm to find the dual prices. However, each time the channel statistics or the user requirements change: (i) the values of the dual prices must be recomputed at the access point; and (ii) the access point has to broadcast these values to the users. Complexity of the algorithm finding the dual prices depends on the number of iterations required until convergence (denoted by N_I) as well as the number of samples used to estimate the expectations in (22)-(25) (denoted by N_E). The second parameter must be taken into account because in most cases a closed-form expression for the expectations is not available, and one has to rely on Monte Carlo runs to obtain these expectations. The computational complexity of the off-line algorithm is then $\mathcal{O}(N_I N_E KMJ)$, with numerical simulations suggesting that in practice $N_E \approx 50-500$ and $N_I \approx 200-2000$ are sufficient.

VI. Numerical Tests

To numerically test our designs, we consider a CR with 1 primary and 3 secondary users, i.e., $J = 4$ and $J_p = 1$. Users transmit with OFDM over $K = 256$ subcarriers that are modulated using uncoded QAM. The BER in each subcarrier can be approximated as $\epsilon_{j,k,m}(p_{j,k,m}, r_{j,m}|h_{j,k}) = 0.2\exp[-p_{j,k,m}h_{j,k}/(2^{r_{j,k,m}}-1)]$ [12]. The power profile considered for the multipath channel corresponds to the test

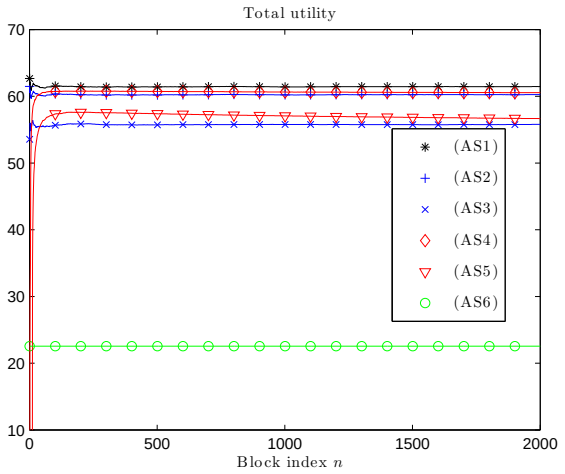


Fig. 1: Utility trajectories (as block index n varies) for different allocation schemes (AS).

channel Vehicular A recommended by the ITU in [39, Table 5], and the average signal-to-noise ratio (SNR) for the different users is set to $6dB$. The AMCP modes are designed so that they correspond to the non-zero uniform random samples of the continuous water-filling solution [20], i.e., $r_{j,k,m} = [\log_2(h/\mu)]_{>0}$ and $p_{j,k,m} = [1/\mu - 1/h]_{>0}$ with the water-filling level $\mu \in [0.05, 20]$, and the channel gain $h \in [0.5, 5\bar{h}_{j,k}]$. Unless otherwise specified, $M_{j,k} = 36$ $\forall(j,k)$. The default utility function for all users is $U_j(\bar{r}_j) = c_j \log(\bar{r}_j)$, with c_j representing a priority constant. The QoS constraints are set to: $\check{\mathbf{r}}^T := [\check{r}_1, \ldots, \check{r}_4] = [40, 15, 15, 40]$ bits per channel use (b.p.c.u), $\check{\mathbf{p}}^T := [\check{p}_1, \ldots, \check{p}_4] = [20, 5, 5, 10]$, and $\check{\epsilon}_j = 0.001$ for all j.

Setting $c_j = 5$ and $\beta_n = (10n)^{-4}$, Figure 1 shows the time evolution of the total utility achieved by six different allocation schemes: (AS1) the benchmark allocation based on P-CSI, $M_{j,k} = \infty$ and the channel CDF assumed known; (AS2) our optimum allocation using $M_{j,k} = 36$ AMCP modes with known channel CDF (Section IV.A); (AS3) the quantized version of (AS2) assuming that the number of regions per subcarrier (i.e., the number of different AMCP modes) is $L_{j,k} = 8$ (Section V.C); (AS4) our optimum allocation using $M_{j,k} = 36$ AMCP, but without knowledge of the channel CDF (Section IV.B); (AS5) the quantized version of (AS4) with $L_{j,k} = 8$; and (AS6) a heuristic allocation that does not adapt subcarriers and power but optimally adapts the transmission rate (P-CSI is assumed at transmitters). The main observation from Figure 1 is that the developed schemes perform very close to the benchmark even for a small-moderate number of AMCP modes. Furthermore, when the P-CSI assumption is not realistic and/or the feedback rate from the access point to the transmitters has to be reduced, the quantized schemes (AS3) and (AS5) that require only $\log_2(L_{j,k}) = 3$ bits of feedback per subcarrier, do not incur a big loss w.r.t. their P-CSI counterparts and perform significantly better than the heuristic allocation. This observation is confirmed by Table II, where the total utility achieved by (AS3) for different values of $L_{j,k}$ is shown. It is worth noting that the gap w.r.t. the

TABLE I: Number of feedback bits and operational complexity (OC) of the on-line algorithms.

Section	Feedback bits	OC at CU	OC at each user
V.A	$B_A = \lceil K \log_2(J+1) \rceil$	$\mathcal{O}(KMJ)$	$\mathcal{O}(MK/J)$
V.B	$B_B = \lceil \sum_{k=1}^{K} \log_2(\sum_{j=0}^{J} M_{j,k}) \rceil$	$\mathcal{O}(KMJ)$	$\mathcal{O}(K)$
V.C	$B_C = \lceil \sum_{k=1}^{K} \log_2(\sum_{j=0}^{J} L_{j,k}) \rceil$	$\mathcal{O}(KMJ)$	$\mathcal{O}(MK/J)$

TABLE II: Total utility achieved for different values of $L_{j,k}$ (AS3).

# of regions	$L_{j,k}=4$	$L_{j,k}=8$	$L_{j,k}=16$	$M_{j,k}=36$	$M_{j,k}=\infty$
# of bits	2	3	4	6	∞
Total utility	51.3	55.7	58.8	60.5	61.8

TABLE III: Final values of average rate and power for different allocation schemes (AS)

AS	$\bar{r}_1$	$\bar{p}_1$	$\bar{r}_2$	$\bar{p}_2$	$\bar{r}_3$	$\bar{p}_3$	$\bar{r}_4$	$\bar{p}_4$
Constraints	40	20	15	5	15	10	40	10
(AS1)	41.6	20.0	15.0	5.1	14.9	6.0	25.0	10.0
(AS2)	40	20.0	14.7	5	15.1	10.1	20.2	10.0
(AS3)	38.1	21.8	8.1	4.9	14.8	10.3	14.4	10.0
(AS4)	40.1	20.1	14.8	5.2	15.2	9.8	20.3	10.1
(AS5)	38	22.1	8.7	5.3	14.9	10.1	15.0	10.0
(AS6)	6.99	20.0	2.2	5.0	2.4	10.0	2.4	10.0

water-filling benchmark could be further reduced if the set $\mathcal{M}_{j,k}$ were optimally designed along the lines of e.g., [25].

To complete the description of the solution, the final values of the average transmitted rate and power for each of the allocation schemes considered are listed in Table III. The results shown validate the proposed resource management approach since the requirements are satisfied: the transmit-power stays below the maximum value for all users, the primary user transmits at a rate higher than its minimum requirement while the transmit-rates of secondary users stay below their maximum allowable levels.

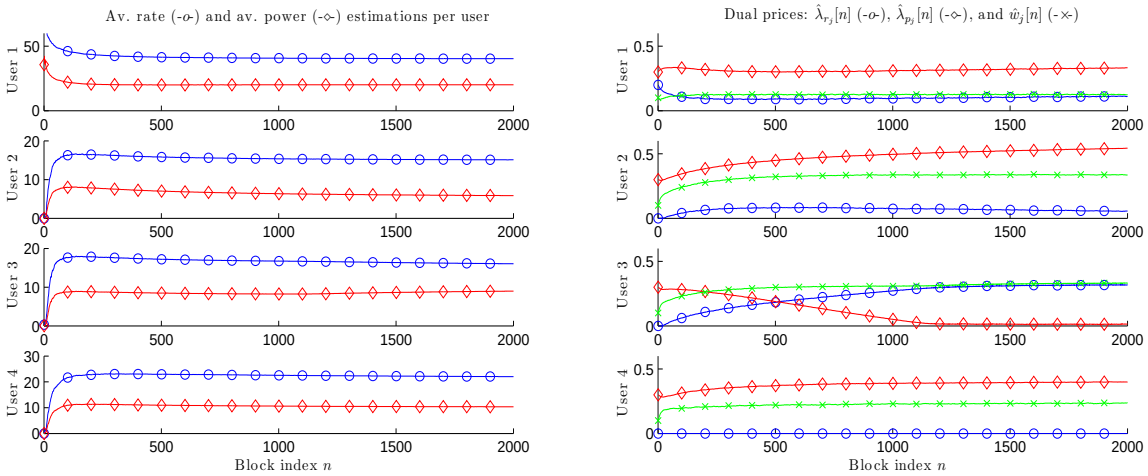
Another interesting observation from Figure 1 is the faster convergence of (AS2) and (AS3) that capitalize on the knowledge of the channel CDF relative to the convergence of their stochastic counterparts (AS4) and (AS5), respectively. This is an expected behavior since (AS4) and (AS5) need time to learn the channel statistics (hundreds of iterations). In addition, convergence may suffer as the number of modes increases. This is because: (i) for high values of $M_{j,k}$, the rate-power relationship among modes can resemble the logarithmic shape of its continuous counterpart; and (ii) if the relationship between power and rate were logarithmic, then the maximum rate constraints in (5) would be non-convex. This in turn could render the subgradient iterations in (22)-(25) possibly non-convergent. Note that since the values of $M_{j,k}$ are always finite, (5) is always convex; but from a numerical perspective, problems may arise for high (yet finite) values of $M_{j,k}$.

To gain more insight on the convergence of our algorithms, Figure 2a plots the trajectories of the sample average of the rate $\hat{\bar{r}}_j[n] := n^{-1} \sum_{q=1}^{n} \sum_{k=1}^{K} r^*_{j,k}[q]$, and sample average of the power $\hat{\bar{p}}_j[n] := n^{-1} \sum_{q=1}^{n} \sum_{k=1}^{K} p^*_{j,k}[q]$ when users implement (AS4); while Figure 2b depicts the corresponding trajectories of the dual prices $\hat{\lambda}_{r_j}[n]$, $\hat{\lambda}_{p_j}[n]$, and $\hat{w}_j[n]$. In both figures each subplot corresponds to a different user.

Simple inspection of Figure 2a reveals that in order to maximize the total utility, the optimal allocation assigns: $\bar{r}_j < \check{r}_j$ and $\bar{p}_j \approx \check{p}_j$ for $j = 2, 4$ (although in both cases users were allowed to transmit at higher rate they do not have enough power); $\bar{r}_1 \approx \check{r}_1$ with $\bar{p}_1 \approx \check{p}_1$ (to ensure that the primary user satisfies its minimum rate requirement, all its power has to be used up); and $\bar{r}_3 \approx \check{r}_3$ and $\bar{p}_3 \lesssim \check{p}_3$ (user 3 has enough power to transmit at higher rate but this would violate its maximum rate constraint).

With reference to Figure 2b, consider the optimal weighs $\hat{w}_j[n]$. The main observation is that the value of $\hat{w}_j[n]$ is higher for the users with smaller average transmit-rate (namely, $j = 2, 3$). This shows that the optimal algorithm tries to promote users who transmit less information, something reasonable since the utility function implemented is $U_j(\bar{r}_j) = c \log(\bar{r}_j)$. Logarithmic utilities are widely used in resource allocation problems because they maximize the overall transmit rate while keeping the rates among users as close as possible. (A popular allocation scheme implementing this class of utilities is the proportional fair scheduling algorithm of [32].)

Moving on to the analysis of dual-prices, numerical results reveal that the behavior depends on the specific simulated user. For the primary user $j = 1$, we observe that since both rate and power constraints are satisfied as equalities, its rate and power Lagrange multipliers take on positive values. However, the role of those multipliers is different. Basically, $\hat{\lambda}_{p_1}[n] > 0$ prevents user 1 from exceeding its power budget (penalizing transmissions over channels entailing a high power consumption). On the other hand, the rate multiplier turns out to be active because $\check{r}_1$ is very demanding. This way, $\hat{\lambda}_{r_1}[n] > 0$ increases the link quality indicator of the primary user so that secondary users may receive lower priority even when they have a good channel realization. For secondary users $j = 2, 4$, it is observed that $\hat{\lambda}_{p_j}[n] > 0$ (i.e., the power constraints are active) while $\hat{\lambda}_{r_j}[n] \approx 0$ (i.e., the rate constraints are slack). This means that when the optimum allocation tries to maximize the total utility, the values of their transmit-rates are below the maximum allowed and there is no need for activating the corresponding Lagrange multipliers. (Notice that, e.g., $\hat{\lambda}_{r_j}[n]$ is not always zero since it is updated based on the instantaneous values of the transmit-rate and



(a) Sample average of individual rate and power

(b) Lagrange multipliers and weights

Fig. 2: Trajectories (as block index n varies) corresponding to (AS4). Each subplot corresponds to a different user.

power [cf. (26)], which implies that although $\hat{\bar{r}}_j[n] > \check{r}_j$, $\sum_{k=1}^{K} r_{j,k}$ can be less than $\check{r}_j$ for some n leading the multiplier to a non-zero value). Finally, for the secondary user $j = 3$, we have $\hat{\lambda}_{r_3}[n] > 0$ with $\hat{\lambda}_{p_3}[n] \approx 0$. In this case, since $\check{p}_3$ is high enough, user 3 is not power-limited but rate-limited (as a secondary user, its rate cannot exceed the maximum value allowed by the CR). Note that although user 3 is rate-limited, its power consumption is also very close to the maximum level. This is because the optimal solution dictates user 3 to transmit over "low-quality" channel realizations that require higher values of transmit-power, so that reliable channel realizations can be used by other (power-limited) users to increase the overall system utility.

Interestingly, we also observe that although the dual-prices converge for all users, the corresponding trajectories are different. Those users with more demanding (selective) constraints exhibit slower convergence. Last but not less important, Table IV shows the final values of the dual-prices corresponding to (AS2) and (AS4) after 2,000 iterations. The listed results confirm that the stochastic iterations (26)-(29) converge to the same point as the average ones in (22)-(25), thus corroborating Theorem 2.

We close this section by presenting the utility trajectories for two variations of the initial test case. In the first scheme, the primary rate constraint $\check{r}_1$ is set to zero (see Figure 3a), while in the second scheme the original utility functions are replaced by $U_j(\bar{r}_j) = c_j(\log(\bar{r}_j) + \bar{r}_j)$; see also Figure 4. From Figure 3a we verify that convergence is faster than in the original constrained case depicted in Figure 1. As pointed out earlier, this slow convergence is due to the fact that the original test case represents a very demanding scenario for which even finding a feasible solution is difficult. Furthermore, since no minimum rate requirement is imposed to the primary user, the new solution achieves a higher total utility level by allowing secondary users to increase their rate. On the other hand, comparison of Figures 1 and 3b reveals that the absolute value of the utility for the new scheme is higher than

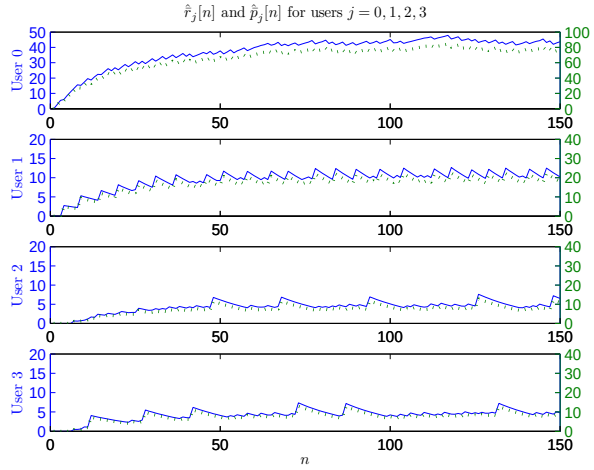


Fig. 4: Utility trajectories (as block index n varies) for different allocation schemes (AS) when $U_j(\bar{r}_j) = c_j(\log(\bar{r}_j) + \bar{r}_j)$ and $\check{r}_1 = 40$.

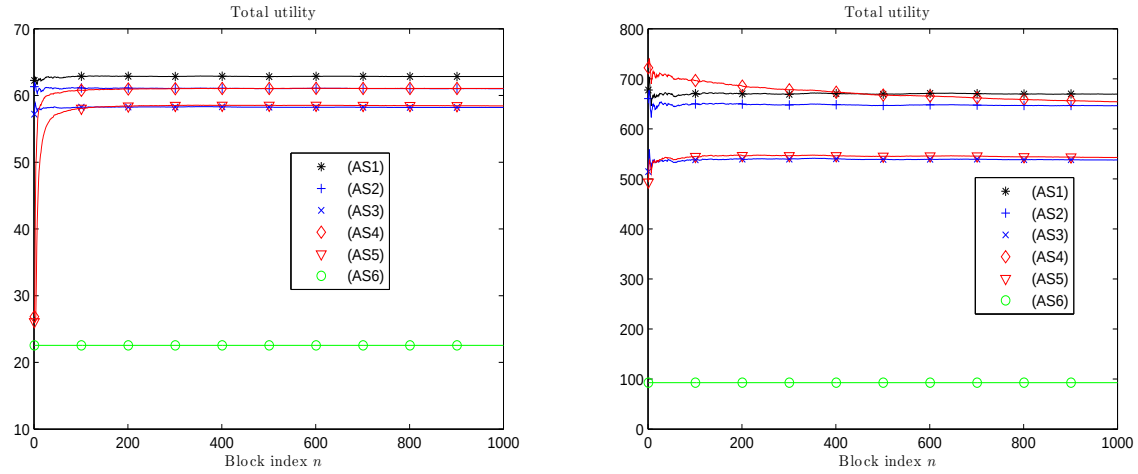
the one obtained before (this was certainly expected since $c_j(\log(\bar{r}_j) + \bar{r}_j) \geq c_j(\log(\bar{r}_j))$; and, the relative utility gap between the developed allocation algorithms and the heuristic scheme is higher for the new test set. This is because the updated utility function is more sensitive to average transmit-rate (its derivative is higher), and therefore suboptimum solutions entail higher penalty.

VII. Conclusions

Taking into account different priorities among users, specific utility functions, individual QoS requirements, and physical layer specifications based on limited-rate feedback, we derived optimal channel-adaptive resource allocation parameters (power, rate, and subcarrier) for OFDMA cognitive radios with a primary-secondary user hierarchy. The resultant optimum resource allocation depends on the current channel realization

TABLE IV: Final values of dual-prices for (AS2) and (AS4).

AS vs. Dual price	λ_{r_1}	λ_{p_1}	w_1	λ_{r_2}	λ_{p_2}	w_2	λ_{r_3}	λ_{p_3}	w_3	λ_{r_4}	λ_{p_4}	w_4
(AS2)	0.112	0.332	0.125	0.055	0.545	0.337	0.320	0.013	0.334	0.000	0.400	0.236
(AS4)	0.111	0.333	0.124	0.055	0.544	0.336	0.321	0.017	0.332	0.000	0.399	0.238



(a) $U_j(\bar{r}_j) = c_j(\log(\bar{r}_j))$ and $\check{r}_1 = 0$

(b) $U_j(\bar{r}_j) = c_j(\log(\bar{r}_j) + \bar{r}_j)$ and $\check{r}_1 = 40$

Fig. 3: Utility trajectories (as block index n varies) for different allocation schemes (AS).

and optimally obtains dual prices. When the channel distribution is known, a subgradient based iterative algorithm was developed to find the optimum dual prices. In addition, when the channel distribution is unknown, a provably convergent stochastic dual algorithm was developed to learn the channel statistics on-the-fly and approach the optimal off-line solution with known channel statistics. Once the values of the dual prices are obtained, the overall optimal solution is fairly simple to implement, and amounts to a greedy-opportunistic access whereby only one user gains access to a given subcarrier per channel realization. Operating conditions were also identified to facilitate partially distributed implementation, and reduce the feedback overhead in both TDD and FDD modes of CR operation. In both cases, the required complexity to implement the novel algorithms as well as the amount of feedback are affordable for most practical systems.[7]

APPENDIX A: DERIVATION OF EQUATION (17)

According to (10), we must maximize the constrained Lagrangian w.r.t. $\boldsymbol{\tau}(\mathbf{h})$, which amounts to solving the convex minimization problem

$$\begin{cases} \min\limits_{\boldsymbol{\tau}(\mathbf{h})} & -L(\boldsymbol{\lambda}, \boldsymbol{\tau}) \\ \text{s.to} & C4.1.\ -\tau_{j,k,m}(\mathbf{h}) \leq 0, \ \ \forall k, m, j, \mathbf{h} \\ & C4.2.\ \sum_{j=1}^{J} \sum_{m \in \mathcal{M}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) - 1 \leq 0, \\ & \qquad\qquad\qquad\qquad \forall k, \mathbf{h}. \end{cases} \tag{31}$$

To deal with the objective in (31), recall that the average rate (power) is a function of $\boldsymbol{\tau}(\mathbf{h})$ since $\bar{r}_j = E_{\mathbf{h}}\left[\sum_{k=1}^{K} \sum_{m \in \mathcal{M}_{j,k}(h_{j,k})} \tau_{j,k,m}(\mathbf{h}) r_{j,k,m}\right]$, and therefore $\partial \bar{r}_j / \partial \tau_{j,k,m} = r_{j,k,m} dF_{\mathbf{h}}(\mathbf{h})$, $\partial \bar{p}_j / \partial \tau_{j,k,m} = p_{j,k,m} dF_{\mathbf{h}}(\mathbf{h})$, and $\partial U_j(\bar{r}_j)/\partial \tau_{j,k,m} = U_j'(\bar{r}_j) r_{j,k,m} dF_{\mathbf{h}}(\mathbf{h})$. Using the expression of $L(\boldsymbol{\lambda}, \boldsymbol{\tau})$ in (9) and the definition of the quality link indicators in (13) and (14), we can use those derivatives to write $\partial - L(\boldsymbol{\lambda}, \boldsymbol{\tau})/\partial \tau_{j,k,m} = -\varphi_{j,k,m}(\mathbf{h}) dF_{\mathbf{h}}(\mathbf{h})$. To account for the constraints in (31), define $\alpha_{\tau_{j,k,m}}(\mathbf{h}) \geq 0$ and $\alpha_{\tau_k}(\mathbf{h}) \geq 0$ as the non-negative Lagrange multipliers associated with $C4.1$ and $C4.2$[8]. Based on the previous expressions, the KKT conditions for the optimum solution of (31) can be written as

$$-\varphi_{j,k,m}(\mathbf{h}) f_{\mathbf{h}}(\mathbf{h}) + \alpha^*_{\tau_k}(\mathbf{h}) - \alpha^*_{\tau_{j,k,m}}(\mathbf{h}) = 0, \quad \forall \mathbf{h}, \ \forall j, \ \forall m \in \mathcal{M}(h_{j,k}); \tag{32}$$

$$\tau^*_{j,k,m}(\mathbf{h}) \alpha^*_{\tau_{j,k,m}}(\mathbf{h}) = 0, \ \forall \mathbf{h}, \ \forall j, \ \forall m \in \mathcal{M}(h_{j,k}); \tag{33}$$

$$\Big[\sum_{j=1}^{J} \sum_{m \in \mathcal{M}(h_{j,k})} \tau^*_{j,k,m}(\mathbf{h}) - 1\Big] \alpha^*_{\tau_k}(\mathbf{h}) = 0, \ \ \forall \mathbf{h}, \ \forall k; \tag{34}$$

where (32) corresponds to setting to zero the partial derivative of the Lagrangian of (31) w.r.t. $\tau_{j,k,m}(\mathbf{h})$, while (33) and (34) are the slackness conditions associated with $C4.1$ and $C4.2$, respectively [5, Sec. 5.5.2]. To find the optimum solution $\tau^*_{k,m,l}(\mathbf{h})$, we need the following lemmas.

Lemma 1: *The solution of (32)-(34) consists of at most one user accessing each subcarrier with a single AMCP mode.*

Proof: Assume that $\tau^*_{j',k,m'}(\mathbf{h}) > 0$ for a specific (j', m') pair. For this pair, (33) implies $\alpha^*_{\tau_{j',k,m'}}(\mathbf{h}) = 0$,

[7]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

[8]The dependence of the multipliers on $\mathbf{h}$ was made explicit since the constraints they correspond to hold for each realization.

which can be substituted into (32) for (j',m') to find that $\alpha^*_{\tau_k}(\mathbf{h}) = \varphi_{j',k,m'}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h})$. Suppose now that $\tau^*_{j'',k,m''}(\mathbf{h}) > 0$ for a pair $(j'',m'') \neq (j',m')$. Repeating the argument, this requires $\alpha^*_{\tau_{j'',k,m''}}(\mathbf{h}) = 0$ and therefore $\alpha^*_{\tau_k}(\mathbf{h}) = \varphi_{j'',k,m''}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h})$; i.e., to have both $\tau^*_{j',k,m'}(\mathbf{h})$ and $\tau^*_{j',k,m'}(\mathbf{h})$ greater than zero, one needs $\varphi_{j',k,m'}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h}) = \varphi_{j'',k,m''}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h})$, which almost surely is not true[9]. Therefore, per subcarrier k, it must hold that $\tau^*_{j,k,m}(\mathbf{h}) > 0$ for no more than a unique pair (j,m). ■

Based on Lemma 1, the next step finds the optimum user-mode pair accessing each subcarrier.

Lemma 2: *The optimal user-mode pair (j_k, m_k) assigned to the kth subcarrier is the one whose subcarrier utility reward is maximum, i.e.,* $(j_k,m_k) := \arg\max_{j,m}\{\varphi_{j,k,m}(\mathbf{h}) : j = 1,\ldots,J,\ m \in \mathcal{M}(h_{j,k})\}$.

Proof: Suppose that $(j'_k,m'_k) \neq (j_k,m_k)$ is the candidate pair to utilize the subchannel k. Using the proof of Lemma 1, this requires $\alpha^*_{\tau_k}(\mathbf{h}) = \varphi_{j'_k,k,m'_k}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h})$. Now writing (32) for the pair (j_k,m_k) yields $-\varphi_{j_k,k,m_k}(\mathbf{h})dF_{\mathbf{h}}(\mathbf{h}) + \alpha^*_{\tau_k}(\mathbf{h}) - \alpha^*_{\tau_{j'_k,k,m'_k}}(\mathbf{h}) = [\varphi_{j'_k,k,m'_k}(\mathbf{h}) - \varphi_{j_k,k,m_k}(\mathbf{h})]dF_{\mathbf{h}}(\mathbf{h}) - \alpha^*_{\tau_{j_k,k,m_k}}(\mathbf{h}) = 0$. Since $\alpha^*_{\tau_{j'_k,k,m'_k}}(\mathbf{h}) \geq 0$, satisfying the latter requires $\varphi_{j'_k,k,m'_k}(\mathbf{h}) \geq \varphi_{j_k,k,m_k}(\mathbf{h})$. This contradicts the definition $(j_k,m_k) := \arg\max_{j,m}\{\varphi_{j,k,m}(\mathbf{h}) : j = 1,\ldots,J,\ m \in \mathcal{M}(h_{j,k})\}$ and proves that the only feasible candidate to use the subcarrier is (j_k,m_k). ■

The optimal resource allocation in (17) follows readily from Lemma 2.

There are two extreme cases where the validity of Lemmas 1 and 2 must be carefully analyzed: (i) when all link quality indicators are non-positive; and (ii) when more than one user attain the maximum value of all link quality indicators. The first event can happen when the channel gains of all users are so poor that the optimal solution has all of them deferring. This is analogous to the optimum single-user power loading dictated by the water-filling algorithm when the channel inverse is so high that no power (water) is allocated to that channel. Lemmas 1 and 2 easily hold for this case since the subcarrier is assigned to the fictitious user $j = 0$ who transmits with zero power and rate (i.e., $\tau^*_{0,k,1}(\mathbf{h}) = 1$).

The second event is unlikely for generic values of $\boldsymbol{\lambda}$ and $\boldsymbol{w}$ but its probability increases when the values of the multipliers are tightly self-adjusted to accurately satisfy the rate and power constraints in (8). In this case, the solution of the Lagrangian allows to implement time policies such that $\tau_{j,k,m}(\mathbf{h}) > 0$ for more than one users. For illustration purposes, consider two users j_1 and j_2 having the same QoS levels, dual prices, AMCP modes and equally favorable channel conditions so that they tie, i.e., their link quality indicators are equal and larger than those of any other user. For this case, if we define $\mathcal{H}^{tie}_k := \{\mathbf{h} : \varphi_{j_1,k,m}(\lambda_{j_1},w_{j_1},h_{j_1}) = \varphi_{j_2,k,m}(\lambda_{j_2},w_{j_2},h_{j_2}) = \varphi_{j^*,k,m^*_{j^*}}(\boldsymbol{\lambda},\boldsymbol{w},\mathbf{h})\}$, then $\tau_{j_1,k,m}(\mathbf{h}) = \tau_{j_2,k,m}(\mathbf{h}) = 1/2$ $\forall \mathbf{h} \in \mathcal{H}^{tie}_k$ is clearly an optimal solution. However, it is also clear that if we split $\mathcal{H}^{tie}_k$ into two disjoint subsets $\mathcal{H}^{j_1}_k$ and $\mathcal{H}^{j_2}_k$ so that the probability of $\mathbf{h}$ belonging to either one of them is the same, the time policy $\tau_{j_1,k,m}(\mathbf{h}) = 1, \tau_{j_2,k,m}(\mathbf{h}) = 0$ if $\mathbf{h} \in \mathcal{H}^{j_1}_k$ and $\tau_{j_1,k,m}(\mathbf{h}) = 0, \tau_{j_2,k,m}(\mathbf{h}) = 1$ if $\mathbf{h} \in \mathcal{H}^{j_2}_k$, is equivalent to the previous one, with the latter belonging to the class covered by Lemmas 1 and 2. The specific value of this probability has to be computed so that the active constraints in (8) are tightly satisfied. It is also worth mentioning that although for mathematical rigor we have shown that Lemmas 1 and 2 hold for the rare cases (i) and (ii), possible suboptimum decisions will lead to small deviations in the QoS requirements; and the probability of these events vanishes exponentially as J, K or $M_{j,k}$ increases. This means that from a performance analysis perspective, they can be ignored in most practical adaptive wireless systems.

[9]Note that $\varphi_{j,k,m}(\mathbf{h})$ depends on the local parameters w_j, λ_{r_j}, λ_{p_j}, $r_{j,k,m}$, $p_{j,k,m}$ as well as on the channel $h_{j,k}$ that corresponds to a realization of a continuous random process.

APPENDIX B: PROOF OF THEOREM 1

Consider the constrained convex optimization problem

$$\begin{cases} \max\limits_{\boldsymbol{\tau}(\mathbf{h}),\mathbf{x}} & \sum_{j=0}^{J} U_j(x_j) \\ \text{s.to} & C1.\ \ \bar{r}_j(\boldsymbol{\tau}(\mathbf{h})) \geq \check{r}_j,\ j = 1,\ldots,J_p \\ & C2.\ \ \bar{r}_j(\boldsymbol{\tau}(\mathbf{h})) \leq \check{r}_j,\ j = J_p+1,\ldots,J \\ & C3.\ \ \bar{p}_j(\boldsymbol{\tau}(\mathbf{h})) \leq \check{p}_j,\ j = 1,\ldots,J \\ & C4.\ \ \boldsymbol{\tau}(\mathbf{h}) \in \mathcal{F}_\tau \\ & C5.\ \ x_j \leq \bar{r}_j(\boldsymbol{\tau}(\mathbf{h})),\ \forall j \end{cases} \tag{35}$$

whose optimal solution solves also the original problem in (8). The only differences between problems (8) and (35) are that (35) includes: (i) an auxiliary variable $\mathbf{x}$ to replace $\bar{r}_j(\boldsymbol{\tau}(\mathbf{h}))$ in the objective; and (ii) constraint $C5$ to enforce that at the optimum both x_j and $\bar{r}_j$ are equal. (Note that since $U_j(x_j)$ is an increasing function of x_j, $C5$ can be written either as a strict equality or as an inequality.) As it will be shown later, the main purpose for introducing $\mathbf{x}$ is to decouple the optimum primal variables, and thus facilitate numerical evaluation of the optimum dual variables.

In this appendix, we will use the notation $\tilde{t}$ to emphasize that the corresponding variable $\tilde{t}$ refers to the problem in (35) and not to the original t in (8). The new constraints $C5$ require defining the Lagrange multipliers $\tilde{\lambda}_{x_j}$ $\forall j$ and incorporating those in the definition of the vector $\tilde{\boldsymbol{\lambda}} := [\tilde{\lambda}_{r_1},\tilde{\lambda}_{p_1},\tilde{\lambda}_{x_1},\ldots,\tilde{\lambda}_{r_J},\tilde{\lambda}_{p_J},\tilde{\lambda}_{x_J}]^T$. Proceeding as in Section III, we can ignore temporarily the instantaneous constraints $C4$, and write the Lagrangian as

$$\begin{aligned} \tilde{L}(\tilde{\boldsymbol{\lambda}},\boldsymbol{\tau},\mathbf{x}) = & \sum_{j=1}^{J} U_j(x_j) + \sum_{j=1}^{J_p} \tilde{\lambda}_{r_j}(\bar{r}_j(\boldsymbol{\tau}) - \check{r}_j) \\ & - \sum_{j=J_p+1}^{J} \tilde{\lambda}_{r_j}(\bar{r}_j(\boldsymbol{\tau}) - \check{r}_j) - \sum_{j=1}^{J} \tilde{\lambda}_{p_j}(\bar{p}_j(\boldsymbol{\tau}) - \check{p}_j) \\ & - \sum_{j=1}^{J} \tilde{\lambda}_{x_j}(x_j - \bar{r}_j(\boldsymbol{\tau})). \end{aligned} \tag{36}$$

The Lagrange dual function is

$$\tilde{D}(\tilde{\boldsymbol{\lambda}}) = \max_{\mathbf{x},\boldsymbol{\tau}\in\mathcal{F}_\tau} \tilde{L}(\tilde{\boldsymbol{\lambda}},\boldsymbol{\tau},\mathbf{x}). \tag{37}$$

Since $\tilde{L}$ is convex in the primal variables, we proceed as in Appendix A using the KKT conditions to find the global optimum w.r.t. $\mathbf{x}$ and $\boldsymbol{\tau}$ as:

$$\tilde{\varphi}_{j,k,m}(\tilde{\boldsymbol{\lambda}},\mathbf{h}) := \left(\tilde{\lambda}_{x_j} + (-1)^{I_{\{j>J_p\}}}\right)\tilde{\lambda}_{r_j} r_{j,k,m} - \tilde{\lambda}_{p_j} p_{j,k,m} \tag{38}$$

$$\tau^*_{j,k,m}(\tilde{\boldsymbol{\lambda}},\mathbf{h}) = \begin{cases} 1, & \text{if } \ (j,m) = \arg\max\limits_{j,m} \tilde{\varphi}_{j,k,m}(\tilde{\boldsymbol{\lambda}},\mathbf{h}) \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

$$x^*_j(\tilde{\boldsymbol{\lambda}}) = U'^{-1}_j(\tilde{\lambda}_{x_j}) = \tilde{r}(\tilde{\lambda}_{x_j}) \tag{40}$$

where $\tilde{r}(\cdot)$ denotes the rate-weight function introduced in Section IV-A.

Substituting (39) and (40) into (37), the dual function is completely characterized. The dual problem of (35) is

$$\min_{\tilde{\boldsymbol{\lambda}}\geq \mathbf{0}} \tilde{D}(\tilde{\boldsymbol{\lambda}}). \tag{41}$$

where due to the convexity of (35) the duality gap is zero. Since the problem in (41) is always convex, the optimum value of the multipliers can be found using the following subgradient iterations:

$$\tilde{\lambda}^{(i+1)}_{r_j} = \left[\tilde{\lambda}^{(i)}_{r_j} - \beta^{(i)}\left(E_{\mathbf{h}}\left[\sum_{k=1}^{K} r^*_{j,k}(\tilde{\boldsymbol{\lambda}}^{(i)},\mathbf{h})\right] - \check{r}_j\right)\right]^+, \quad j = 1,\ldots,J_p \tag{42}$$

$$\tilde{\lambda}^{(i+1)}_{r_j} = \left[\tilde{\lambda}^{(i)}_{r_j} - \beta^{(i)}\left(\check{r}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} r^*_{j,k}(\tilde{\boldsymbol{\lambda}}^{(i)},\mathbf{h})\right]\right)\right]^+, \quad j = J_p+1,\ldots,J \tag{43}$$

$$\tilde{\lambda}^{(i+1)}_{p_j} = \left[\tilde{\lambda}^{(i)}_{p_j} - \beta^{(i)}\left(\check{p}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} p^*_{j,k}(\tilde{\boldsymbol{\lambda}}^{(i)},\mathbf{h})\right]\right)\right]^+, \quad j = 1,\ldots,J \tag{44}$$

$$\tilde{\lambda}^{(i+1)}_{x_j} = \left[\tilde{\lambda}^{(i)}_{x_j} - \beta^{(i)}(x^*_j(\tilde{\boldsymbol{\lambda}}^{(i)}) - \bar{r}_j(\boldsymbol{\tau}^*(\mathbf{h}))\right]^+, \quad j = 1,\ldots,J \tag{45}$$

with stepsize $\beta^{(i)} \downarrow 0$. By identifying $\lambda_{r_j} = \tilde{\lambda}_{r_j}$, $\lambda_{p_j} = \tilde{\lambda}_{p_j}$, and $w_j = \tilde{\lambda}_{x_j}\ \forall j$, it is easy to see that (42)-(45) correspond to the iterations (22)-(25). Moreover, convexity of (41) implies [4, Sec. 6.2] that the subgradient iterations exhibit linear convergence as claimed in Theorem 1.

APPENDIX C: PROOF OF THEOREM 2

To prove the wanted convergence, it is first useful to recognize that the dual variable updates in (26)-(29) and those in (22)-(25) can be seen as a pair of primary and averaged systems [29, Chapter 7]. This is because the updates in the former follow a stochastic subgradient direction, which is an unbiased "instantaneous" estimate of the subgradient direction used in the latter; i.e., taking expectation over fading realizations $\mathbf{h}$ of the stochastic subgradient used in (26)-(29) yields the subgradient used in (22)-(25). Relying on stochastic approximation tools, we can then show that trajectories of these two (primary and averaged) systems are close to each other under regularity conditions.

The dual variable updates in (26)-(29) have similar forms with the queue size updates of the greedy primal-dual (GPD) algorithm in [30]. Following the fluid-limit approach detailed in [30], we can define a fluid path (indexed by t) such that $\lambda_{r_j}(t) = \hat{\lambda}_{r_j}[t\beta^{-1}]$, $\lambda_{p_j}(t) = \hat{\lambda}_{p_j}[t\beta^{-1}]$, and $\lambda_{w_j}(t) = \hat{\lambda}_{w_j}[t\beta^{-1}]$. As $\beta \to 0$ and upon letting $n = t\beta^{-1}$, we can rewrite (26)-(29) as

$$\frac{\lambda_{r_j}(t+\beta)-\lambda_{r_j}(t)}{\beta} = -\left(\sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h}) - \check{r}_j\right), \quad j = 1,\ldots,J_p \tag{46}$$

$$\frac{\lambda_{r_j}(t+\beta)-\lambda_{r_j}(t)}{\beta} = -\left(\check{r}_j - \sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right), \quad j = J_p+1,\ldots,J \tag{47}$$

$$\frac{\lambda_{p_j}(t+\beta)-\lambda_{p_j}(t)}{\beta} = -\left(\check{p}_j - \sum_{k=1}^{K} p^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right), \quad j = 1,\ldots,J \tag{48}$$

$$\frac{w_j(t+\beta)-w_j(t)}{\beta} = -\left(\tilde{r}_j\left(\boldsymbol{w}(t)\right) - \sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right), \quad j = 1,\ldots,J. \tag{49}$$

From this fluid path argument with $\beta \to 0$, consider a continuous-time (indexed by t) fluid sample path (FSP), whose evolution satisfies the ordinary differential equations (ODEs) (cf. [30, Lemma 19]):

$$\frac{d\lambda_{r_j}(t)}{dt} = -\left(E_{\mathbf{h}}\left[\sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right] - \check{r}_j\right), \quad j = 1,\ldots,J_p \tag{50}$$

$$\frac{d\lambda_{r_j}(t)}{dt} = -\left(\check{r}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right]\right), \quad j = J_p+1,\ldots,J \tag{51}$$

$$\frac{d\lambda_{p_j}(t)}{dt} = -\left(\check{p}_j - E_{\mathbf{h}}\left[\sum_{k=1}^{K} p^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right]\right), \quad j = 1,\ldots,J, \tag{52}$$

$$\frac{dw_j(t)}{dt} = -\left(\tilde{r}_j\left(\boldsymbol{w}(t)\right) - E_{\mathbf{h}}\left[\sum_{k=1}^{K} r^*_{j,k}(\boldsymbol{\lambda}(t),\boldsymbol{w}(t),\mathbf{h})\right]\right), \quad j = 1,\ldots,J. \tag{53}$$

Interestingly, for stationary and ergodic wireless channels and bounded transmit-powers and rates, it can be shown that (cf. [30, Theorem 3]):

Lemma 3: *The trajectory of the updates (26)-(29) converges in probability to that of the corresponding FSP satisfying (50)-(53) as* $\beta \to 0$.

On the other hand, the ODEs (50)-(53) also describe the updates in the averaged system (22)-(25) as $\beta \to 0$. Since Theorem 1 guarantees convergence of the latter to $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$, if follows readily see that the trajectory of the FSP satisfying (50)-(53) converges to $\boldsymbol{\lambda}^*$ and $\boldsymbol{w}^*$ too. The theorem follows from this fact together with Lemma 3.

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