Evaluating temporary retail price discounts using semiparametric regression

M.P. Martínez-Ruiz and A. Mollá-Descals
Department of Marketing, University of Valencia, Valencia, Spain

M.A. Gómez-Borja
Department of Marketing, University of Castilla-La Mancha, Castilla-La Mancha, Spain, and

J.L. Rojo-Alvarez
Department of Signal Theory and Communications, University Carlos III of Madrid, Madrid, Spain

Abstract
Purpose – To analyze the impact of temporary retail price discount on a consumer goods product category using semiparametric regression and considering different promotional price discount characteristics as well as brand characteristics.

Design/methodology/approach – A semiparametric regression model using Support Vector Machines, which aim to evaluate retailers’ decisions about temporary price discounts, has been developed. The model is derived from the analysis of historical sales data, which provide precise evaluation of previous temporary price discounts periods. The model is also consistent with ample empirical evidence showing that historical retail sales data can be used to evaluate the impact of past promotions.

Findings – Provides an estimation of the shape of the deal effect curve, indicating which temporary price discounts are more effective to increase sales and showing the existence of different threshold and saturation levels. Confirms that promotional price discounts accelerate sales especially during week ends. Evidences that promoting high-priced (high-quality) brands has a stronger impact on sales of low-priced (low-quality) brands than the reverse and that cross-price effects are stronger on the sales of brands with similar prices. Suggests the convenience of the use of the proposed semiparametric methodology to the study of the promotional effects considered.

Research limitations/implications – It is not possible to generalize the modelled shapes of the deal effect curves. There is no information available on feature advertising nor displays. It is important to determine the generalizability of these results to the study of additional promotional effects. It would also be interesting to assume that the retailer’s deal policy is exogenous.

Originality/value – Provides a relevant tool to assess the set of price promotional periods by the grocery retailer. With a more precise and accurate knowledge about the performance of past temporary price cuts, retailers can implement more effective promotional periods.

Keywords Retailing, Discounts, Price positioning, Promotional methods

Paper type Research paper

Introduction
Price promotion plays a role of increasing importance in current markets. Special emphasis on price promotions is pronounced in the grocery industry, where retailers frequently use temporary price discounts to lure customers into stores and boost sales. Many studies reflect that the use of promotional price cuts has increased over the years in retailing and ample empirical evidence supports the use of temporary price reductions to increase sales (e.g. Cotton and Bab, 1978; Wilkinson et al., 1982; Walters and MacKenzie, 1988; Currim and Schneider, 1991; Blattberg et al., 1995; Van Heerde et al., 2001).

Given the increased focus of retailers on temporary price discounts, the measurement and understanding of the associated effects becomes a relevant research topic. Retailers spend an increasing amount of money on temporary price reductions and the management of this promotional activity can be improved through careful measurement of its effects. With superior knowledge of these effects, retailers can improve their decisions about

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Evaluating temporary retail price discounts

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The relationship between temporary price discounts and sales can be addressed by estimating the deal effect curve. This is a topic of great relevance as it determines the optimal dealing amount: if the curve is convex (i.e., has increasing returns), the retailer will be able to run deeper deals than if the effect is concave (i.e., has decreasing returns). However, as this is an issue with limited empirical results, little is known about the shape of this curve. Therefore, this is still an open question for which additional research is needed (Blattberg et al., 1995).

Many phenomena are related to the shape of the deal effect curve: threshold effects, saturation effects, cross-item deal effects, interaction effects between deals of different items, and interaction effects between deals and other types of promotions (Blattberg and Neslin, 1990). While threshold effects show the minimum value of a promotional price discount required to change consumers purchase intentions, saturation effects indicate the limit in the amount consumers can stockpile and/or consume in response to a promotional discount (Blattberg et al., 1995). Temporary price discounts can generate cross-brand competitive effects as purchases of products in one brand of the category may lead to decreases in sales of substitute products in the same category (Sethuraman, 1996; Sethuraman et al., 1999). Though previous research does not address how the shape of the deal curve depends on the simultaneous presence and amount of other items discounts, the effect of display and feature advertising was analyzed by Woodside and Waddle (1975), Blattberg and Wisniewski (1989), Kumar and Leone (1988) and Bemmaor and Mouchoux (1991). However, additional research is needed to understand the complexities of the potential synergies among temporary price discounts, features and displays.

Regarding all these phenomena, we attempt to evaluate the impact of temporary price discounts periods characteristics on category sales considering different brand tiers within the category analysed. Whereas the magnitude of relative discounts and the day of the week of each promotional day will be the temporary price discounts periods characteristics considered, the positioning of brands in terms of the price tier will be included as brand characteristic.

We assume that it is important to include the magnitude of relative discounts as predictor of promotional sales because many previous works have considered that perceptions of deal prices and discounts determine consumer purchase decisions, being the role of deal price perceptions illustrated in several marketing models (Blattberg and Neslin, 1990). The influence of these perceptions has been differently operationalized in the marketing models. It has sometimes been operationalized as regular prices (Blattberg and Wisniewski, 1989; Wittink et al., 1987; Gupta, 1988; Bronnenberg and Watthieu, 1996; Christen et al., 1997; Foekens et al., 1999), actual prices (Kumar and Leone, 1988; Blattberg and Wisniewski, 1989; Krishnamurthi et al., 1992; Sethuraman, 1996; Foekens et al., 1999), net prices (Papatla and Krishnamurthi, 1996), relative deal discounts (Blattberg and Wisniewski, 1989), as absolute deal discounts (Gupta, 1988) or price indices (Wittink et al., 1987; Wittink et al., 1988; Van Heerde et al., 2001). The inclusion of deal price perceptions makes it possible to observe how sales respond to different deal magnitudes and to identify the discount levels for which temporary price discounts are most effective.

Conceptual framework

Most researchers and retailers agree that the primary function of temporary retail price discounts is to increase retailer sales and in turn, retailer profit (e.g., Davidson et al., 1988; Walters and MacKenzie, 1988; Kumar and Leone, 1988; Blattberg and Neslin, 1990; Walters, 1991). An extensive body of research has examined the consumer response to temporary price cuts (e.g., Currin and Schneider, 1991; Ailawadi and Neslin, 1998) and the marketing literature confirms that temporary retail price discounts generate substantial short-term sales increases, showing that this promotional variable under retailers’ control is effective in temporarily increasing sales of the promoted brand (Blattberg et al., 1995).
Another critical aspect of temporary price discounts intervals relates to the days of the week that compound the promotional period. Given that sales show a growing pattern during weekends in grocery retailing, it is important to analyse whether this growing pattern is different during promotional periods, and in this case, to quantify the magnitude of the sales increase during weekend promotional periods. Then, the explicit consideration of the day of the week within the promotional period can be highly informative. In particular, it allows retailers to observe whether a price deal may reinforce sales obtained during weekends. The development of studies including the day of the week among the characteristics of price discounts may allow researchers to gain a greater understanding about the promotional days that represent the most important contribution to the sales spike.

Finally, we also include the positioning of brands in terms of price tiers. Since grocery stores typically discount different brands in the same category simultaneously, temporary price discounts can generate cross-brand competitive effects, being observed that promoting one brand may have negative effects on sales of other competitive brands in the product category (Kumar and Leone, 1988; Walters, 1991). This outcome can happen because different types of products offered in the store can satisfy a specific consumer need, differing only slightly in price and other features (Walters, 1991). Prior research also indicates that temporary price discount effectiveness is asymmetric for low-priced brands compared to high-priced brands, implying that low-priced brands have a more limited price promotional influence relative to high-priced brands (Blattberg and Wisniewski, 1989; Sethuraman, 1996; Sethuraman et al., 1999). Thus, if promoted brands cause extensive substitution effects, the retailer may not be better off as a result of the promotion. In fact, the retailer may even be worse off if consumers switch purchases from high-margin unpromoted brands to low-margin promoted ones. If the sales of nonpromoted brands are unaffected by promotion of a given brand, retailer income will be positively affected.

### Data and methodology

#### Database description
The database used in this research was obtained from a Spanish supermarket. The raw data consist of information on daily sales collected with in-store EAN scanning equipment during one year, 1999 (304 days of sales information). From the database we can draw information about regular prices, promotional prices and promotional periods of all product categories sold by the retailer. There is no information available on feature advertising nor display activity. We focused our research on one category, ground coffee, and selected one packaged size (250 grams) on the basis of the following criteria: the existence of daily sales records and frequent promotions for all the brands of the category and the fact that this category has been frequently used in many previous studies (e.g. Gupta, 1988). Six brands are sold in this category: four high-priced brands (Saimaza, Soley, Marcilla and Bonka) and two low-priced brands (Bahia and 154). There is no store brand.

#### Methodology, model specification and variable operationalization
We built up a model of promotional sales as a possibly complex function of a number of predictor variables. Given the characteristics of our database, we propose the use of a semiparametric regression model because it has the advantages of non-parametric regression (flexibility) for the deal variables, the advantages of parametric regression (efficiency) for indicator variables (Robinson, 1988; Van Heerde et al., 2001) and reduces the incidence of heteroscedasticity as well as serial correlation (Ruppert et al., 2003). Although semiparametric approaches have been shown to be superior both to parametric and non-parametric models, they still exhibit some degree of overfitting in the estimation sample when compared to the validation sample. In this paper we analyse several previously proposed predictors together with a daily temporal description. This formulation becomes unfeasible in terms of number of features supported either by parametric or by recently proposed semiparametric methods, so that we propose a new and robust semiparametric method based on the Support Vector Statistical Methodology.

In particular, we develop a semiparametric regression model using Support Vector Machines to evaluate retailers’ decisions about temporary price discounts. This model is derived from the analysis of historical sales data, which provide precise evaluation of previous temporary price discounts periods. This model is also consistent with ample empirical evidence showing that historical retail sales data can be used to evaluate the impact of past promotions (e.g. Moriarty, 1985; Dhebar et al., 1987; Van Heerde et al., 2001). As we have scanner data records available on a daily basis, we are able to analyse the influence of the day of the week of the promotional days on this phenomenon. Therefore, we can determine which days of the promotional period sales increase more.

The known robustness of the non-parametric support vector method, when formulated as a semiparametric procedure, makes it feasible to analyze an increased number of exogenous variables, thus allowing a more detailed time description in the daily time series available. In this SVM-SR model we express brand sales as the sum of a non-parametric function of metric variables (capturing non linear complex interactions among the metric variables) and a parametric function of other dichotomic predictors. Moreover, SVM-SR has exhibited in our experiments excellent robustness properties in some of our models with moderate multicollinearity.

The specification of the model SVM-SR is shown in the Appendix. The method consists of a description of the solution in terms of some relevant and necessary observations, the Support Vectors, which contain all the necessary information relevant for the model. The use of a composed Mercer’s kernel allows us to state a semiparametric regression model by using a radial basis function kernel for dealing with the non-parametric modelled variables, and a linear kernel for dealing with the dichotomic variables. The advantages of this procedure with respect to other SR schemes can be seen in this setting as mainly numeric, this is, the SVM-SR model is built by solving a quadratic programming problem which warrants that a single minimum exists; no experiment is necessary for fixing the free parameters as simple cross-validation is always enough; not all the samples are involved in building the solution, but only a well-selected subset of them;
it is robust when faced to multicollineality, and it is a well regularized approach. An excellent introduction to the SVM principles and properties can be found in Vapnik (1998).

Model estimation and results

Fitness and validity

For comparison purposes, we provide comparisons with the specific methodologies that to the date have devoted to the study of the effectiveness of temporary price discounts. For this reason, a fully parametric (linear) model using ordinary least squares (OLS) is also adjusted to the data, considering both additive and multiplicative effects by taking the natural logarithm of the sale units as well as the actual prices. Also, a semiparametric standard model (Van Heerde et al. 2001) and a SVM-SR multiplicative model are provided. Therefore, four additional models were adjusted to each brand data: parametric (linear) additive and multiplicative, standard semiparametric and SVM-SR additive. Each time series was split into training (75 percent) and test (25 percent) samples. The $R^2$ and the conventional root mean squared error (RMSE) were obtained in each model for both training and test subsets, in order to be able to detect the presence of overfitting. Confidence intervals (CI) were calculated for linear coefficients (95 percent level).

Table I and Table II show the fitness and predictive validity statistics for all models.

In general, higher values of $R^2$ (and lower of RMSE) can be observed in SVM-SR models. $R^2$ values fall at test observations in most brands but RMSE values mostly hold at test observations, thus indicating that we can expect low level of overfitting and that the models have in fact captured around 60 percent-80 percent of the variability.

Analysis of the influence of the magnitude of relative discounts

All own-item deal effect curves show an inverse relationship between price discounts and sales increases, indicating that the temporary price discounts offered in the category generated sales increases. Own-item actual prices are at the $x$-axis and predicted increase in sales units are at the $y$-axis. As an example, one own-item deal effect curve is shown in Figure 1.

A higher sales response is detected in the high-priced brands of the category. The largest promotional spikes are obtained in Soley, and the smallest, in Bahía. This finding is consistent with the marketing literature, which states that high-priced brands may capture a large proportion of switchers (Blattberg et al., 1995).

In the estimated curves convex and reverse S shapes can be detected, which indicate the existence of different threshold and saturation levels. Whereas the threshold levels obtained vary between 4 percent and 8 percent levels of discount, saturation levels vary between 14 percent and 20 percent levels of discount. Saturation levels indicate the limit in the

<table>
<thead>
<tr>
<th>Table I</th>
<th>Fitness and predictive validity statistics for high-priced brands</th>
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</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>RMSE</td>
</tr>
<tr>
<td>Train</td>
<td>Test</td>
</tr>
<tr>
<td>Marcilla</td>
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</tr>
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</tr>
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</tr>
<tr>
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<td>Bonka</td>
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<th>Fitness and predictive validity statistics for low-priced brands</th>
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<td>Test</td>
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<tr>
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<td>154</td>
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<td>SVM mul.</td>
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amount consumers can stockpile due to the promotion and threshold levels can be explained by consumers perceptions of deals primarily (Blattberg et al., 1995).

Analysis of the influence of the day of the week
The effect of the day of the week in the promotional sales spike is captured by the coefficient estimators of the two first groups of dummy variables, corresponding to the day of the week during promotional periods and to the day of the week during non-promotional periods. For almost all brands of the category but a low-priced brand, Bahía, the weekend in promotional periods have a positive and differential impact over sales. The most important influence of the weekend in promotional periods corresponds to high-priced brands. Also, in almost all high-priced brands (Marcilla, Bonka and Saimaza) the positive influence of the weekend on sales can be detected even on Thursday. Figure 2 shows the estimations obtained for one brand.

Analysis of the positioning of brands in terms of the brand tier
Figure 3 presents one example of the three-dimensional deal effect surfaces that is depicted to illustrate cross-price promotional effects and interaction effects between price discounts of different items. The vertical axis represents the predicted sales volume of the own-item, and the horizontal axis represents the actual prices of the own-brand and of a competing brand. The effect of the remaining brand is cancelled in the model for each of these representations. In order to obtain reliable surfaces, observations with simultaneous price discounts for two or more items are required. Marcilla, Saimaza, and Bonka are the brands with more simultaneous price cuts observations. No doubt that consumer perceptions of simultaneous deal discounts are also important determinants of the results obtained in the three dimensional surfaces.

The interpretation of different curves within each surface is important in order to address cross-promotional effects. The curve A-B represents the own-item deal effect curve when the competing brand is not sold on promotion. The curve B-C shows the changes in sales of the own brand as a response to the deals offered in the competing brand, being the own-item sold with the maximum discount. The curve C-D represents the own-item deal effect curve when the competing brand is sold at the maximum discount. The curve D-A shows the changes in sales of the own-item as a response to the deals offered in the competing brand, being the own-item sold with the maximum discount.

In general, it has been observed that: curves A-B tend to increase in magnitude with the decreases in own brand price; curves B-C tend to increase in magnitude with the increases in competing brand price; curves C-D tend to increase in magnitude with the decreases in own brand price and finally, curves D-A tend to increase in magnitude with the increases in competing brand price. These findings suggest that the interaction effects are asymmetric, thus supporting an empirical generalization in the sales promotion field: “cross-promotional effects are asymmetric and promoting higher quality brands impacts weaker brands disproportionately” (Blattberg et al., 1995, p. G124). In addition to the asymmetric cross-promotional effect, we can also consider the neighborhood cross-price effect (Sethuraman et al., 1999), which states that brands that are closer to each other in terms of price, have larger cross-price effects than brands which are priced further apart. This effect also applies as we can observe the largest cross-deal discount effects on brands that are close to each other in price.

Conclusions, implications and directions for further research
In this paper we have studied the impact of temporary retail price discounts characteristics and the positioning of brands in terms of the brand tier on category sales. Whereas the magnitude of discounts and the days of the week that compound the promotional period have been considered as temporary price discounts characteristics, the positioning of brands in terms of the price tier has been included as brand characteristic. The consideration of daily data has provided a more detailed understanding of the phenomena involved, and it has allowed us to measure the influence of the day of the week of promotional days in brand sales.

From this work several theoretical implications can be derived. In the first place, we have estimated the deal effect
curve with a semiparametric regression model using Support Vector Machines – the SVM-SR model. This model is easy to use and provides a regression analysis that accommodates flexible interaction effects for the price discounts of different brands in the non-parametric component. When compared to standard parametric and semiparametric models, the SVM-SR model exhibits a better fit in our data.

Secondly, the shape of the deal effect curves obtained has shown how temporary price discounts are more effective to increase sales in the high-priced brands of the category and provides empirical support for threshold and saturation levels. In particular, the existence of several threshold and saturation levels is detected in various brands of the category.

Furthermore, it is confirmed that promotional price discounts accelerate sales especially during week ends, this acceleration effect being more significant in the high-priced brands of the category. Our results are also consistent with the existence of asymmetric cross-price effects and neighborhood cross-price effects. Then, it is noted that promoting high-priced (high-quality) brands has a stronger impact on sales of low-priced (low-quality) brands than the reverse and that cross-price effects are stronger on the sales of brands with similar prices.

The use of a regression-type model has provided our model with easy-to-interpret coefficients, at the expense of a loss in terms of the time-series structure of the data. Theoretical effort should be made to create a model with the best characteristics of each approach. Also, clear cut-off statistical tests are to be developed for SVM approaches in general, and for SVM-SR in particular, which will increase the informative power of the method.

This research also implies interesting managerial implications. In particular, this work provides a relevant tool to assess the set of promotional periods by the grocery retailer. With a more precise and accurate knowledge about the performance of past temporary price discounts periods, retailers can implement more effective promotional periods.

Further research is needed to determine the generalizability of the modelled shape of the deal effect curves. The availability of data referred to the usage of feature advertising and displays must lead us to the study of additional promotional effects. It is also important to determine the generalizability of these results to other category products and package sizes, as well as to the study of additional promotional effects, such as complementary effects in other categories. It would be also interesting to assume that the retailer’s deal policy is exogenous.

Note

1 All prices are expressed in pesetas, the Spanish currency during the consideration period. 1€ = 166.386 pesetas

References


Further reading


Appendix

For $k$th item, $k = 1, \ldots, J$, the model is expressed by two different sets of weights, one for the dichotomic variables and another for the metric variables:

$$y_n = \sum_{m=1}^{M} \alpha_{nm}^* x_n^m + \sum_{d=1}^{D} \beta_{nd}^d x_n^d + e_n$$

where:

$y_n$ is the unite sales of brand $k$ in day $n$, $n = 1, \ldots, T$;

$x_n^m$ is the $m$th metric variable value ($m = 1, \ldots, M$) in day $n$;

$x_n^d$ is the $d$th dichotomic variable value ($d = 1, \ldots, D$) in day $n$;

$e_n$ is the disturbance term.

The problem can be stated (Vapnik, 1998) as the minimization of:

$$L_p = \frac{1}{2} \sum_{m=1}^{M} (\alpha_{nm})^2 + \frac{1}{2} \sum_{d=1}^{D} (\beta_{nd})^2 + C \sum_{n=1}^{N} (\xi_n + \xi_n^*)$$

constrained to:

$$y_n - \sum_{m=1}^{M} \alpha_{nm}^* x_n^m - \sum_{d=1}^{D} \beta_{nd}^d x_n^d \leq e + \xi_n$$

$$-y_n + \sum_{m=1}^{M} \alpha_{nm}^* x_n^m + \sum_{d=1}^{D} \beta_{nd}^d x_n^d \leq e + \xi_n^*$$

$$\xi_n, \xi_n^* \geq 0$$

By stating the Lagrange functional for this problem, and then obtaining the gradient with respect to the weight variables, we have:

$$\nu_m = \sum_{n=1}^{N} (\alpha_{nm} - \alpha_{nm}^*) x_n^m$$

$$\nu_d = \sum_{n=1}^{N} (\alpha_{nd} - \alpha_{nd}^*) x_n^d$$

where $\alpha_{nm}, \alpha_{nm}^*$ are the Lagrange multipliers corresponding to (3), (4), and hence, the weights are expressed as a linear combination of the observations. Only a subset of the observations has a nonzero coefficient in (5), (6). They are called Support Vectors, and the solution is fully expressed in terms of them. Dual functional can be stated as the maximization of:

$$L_D = -\frac{1}{2} \sum_{n,k} \left( \alpha_n - \alpha_n^* \right) \left( \alpha_k - \alpha_k^* \right)$$

$$+ \sum_{n=1}^{M} \alpha_{nm}^* x_n^m \sum_{d=1}^{D} \beta_{nd}^d x_n^d - e \sum_{n=1}^{N} \left( \alpha_n + \alpha_n^* \right)$$

$$+ \sum_{n=1}^{N} \left( \alpha_n - \alpha_n^* \right) y_n$$

and the regression function can be expressed as:

$$\hat{y}_n = \sum_{m=1}^{M} \alpha_{nm}^* x_n^m + \sum_{d=1}^{D} \beta_{nd}^d x_n^d$$

This represents a parametric (linear) model for both metric and dichotomic variables. However, it is possible to use Mercer’s kernels in order to introduce a non-parametric (nonlinear) relationship for the metric variables, and after some manipulations frequently used in the SVM methodology (Vapnik, 1998), the regression function can be more conveniently expressed as:

$$\hat{y}_n = \sum_{n=1}^{N} \left( \alpha_n - \alpha_n^* \right) K(x_n^m, x_n^m) + \sum_{d=1}^{D} \beta_{nd}^d x_n^d$$

Confidence intervals (CI) for the parametric part of the models can be readily calculated for linear coefficients (95 percent level) using bootstrap resampling.
The model specification contains the most relevant variables involved in the promotional and nonpromotional periods. Whereas the actual price of each brand will allow to include complex interaction effects among brands in terms of relative discounts, the consideration of the weekly nature of oscillations will introduce temporal information in the model. We employ a semiparametric model in which we express sales units as the sum of a non-parametric function of actual prices and a parametric function of dichotomic variables, specifically, day-of-week variables:

\[
\hat{y}_t = \sum_{n=1}^{N} (\alpha_n - \alpha_n^*) K((x_{nm}^m), (x_{mn}^m)) + \sum_{d=1}^{D} x_t^d \theta_d
\]  

Therefore, two different exogenous (indicator) sets of variables are introduced in our model. First, six metric variables contain the actual prices of all the brands in the category. Second, twelve dummy or indicator variables indicate the day of the week of promotional periods, and the day of the week of non-promotional periods. The use of different sets of dummy variables allows us to model different daily patterns for promotional and for nonpromotional periods:

- \(x_{t1}^d, \ldots, x_{t6}^d\): indicators of the day of week – Monday to Saturday – during promotional periods in brand \((k)\);
- \(x_{t7}^d, \ldots, x_{t12}^d\): indicators of the day of week – Monday to Saturday – during non-promotional periods in brand \((k)\).

This model fulfills the requirement of metric variables allowed to model possibly complex interactions (non-parametric part of the model). The qualitative indicator variables are retained by the linear part of the model, which makes their effect very easy to examine. Also, no interaction effects are to be produced among different days of the week and hence they do not need to be modelled non-parametrically.

**Corresponding author**

MP. Martínez-Ruiz can be contacted at: M.Pilar.Martinez-Ruiz@uv.es