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A MILP-based heuristic algorithm for transmission expansion planning problems

Phillipe Vilaça [a,*], Alexandre Street [b], J. Manuel Colmenar [a]

[a] GRAFO, Universidad Rey Juan Carlos, Móstoles, Spain
[b] LAMPS, DEE, PUC-Rio, Rio de Janeiro, Brazil

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A B S T R A C T

In the last years, a lot of effort was placed into approximated or relaxed models and heuristic and metaheuristic algorithms to solve complex problems, mainly with non-linear and non-convex natures, in a reasonable time. On one hand, approximated/relaxed mathematical models often provide convergence guarantees and allow the problem to be solved to global optimality. On the other hand, there is no guarantee that the optimal solution of the modified problem is even feasible in the original one. In contrast with that, the metaheuristic algorithms lack mathematical proof for optimality, but as the obtained solutions can be tested against the original problem, the feasibility can be ensured. In this sense, this work brings a new method combining exact solutions from a Mixed-Integer-Linear-Problem (MILP) Transmission Expansion Planning (TEP) model and stochastic solutions from metaheuristic algorithms to solve the non-linear and non-convex TEP problem. We identify the issues that came up with the linear approximations and metaheuristics procedures and we introduce a MILP-Based Heuristic (MBH) algorithm to overcome these issues. We demonstrate our method on a single-stage TEP with the RTS 24 nodes and on a multi-stage TEP with the IEEE 118 nodes test system. The AC TEP solution was obtained using Evolutionary Computation, while the DC TEP solution was obtained using a commercial solver. From the simulations results, the novel MBH method was able to reduce in 42% and in 85% the investment cost from an evolutionary computation solution for the single-stage and multi-stage TEP, respectively.

Introduction

a. *Motivation and background*

The transmission grid is an essential part of power systems, it allows not only the physical connection between generators and load centers but also the interconnection between areas, enabling the portfolio effect in which generators with different characteristics can be integrated. Even with the unbundling and liberalization of the electric sector in which the generation and retail sectors were exposed to market mechanisms, the transmission network is considered a natural monopoly, and the studies regarding their evolution over time are frequently conducted in a way that the security of supply and the investment in new assets are both optimized. Therefore, the Transmission Expansion Planning (TEP) aims at identifying a set of assets, as overhead lines, submarine cables, and transformers, to be installed on the grid considering a long-term planning horizon.

The mathematical model representing the TEP problem considers the investment cost in new pieces of equipment and their impact on the system's future operation. For the operation conditions prediction, an AC Optimal Power Flow (AC-OPF) is the model that represents more precisely the intrinsic characteristics of the grid infrastructure, thus, TEP model has non-linear and non-convex natures which make the problem intractable on the commercial solvers. Due to the mentioned importance of the transmission grids for power systems, and the fact that TEP is a capital-intensive exercise (billions of dollars per year), academy and power industry often work with the weld spread DC-OPF model, which approximates the equations from the AC-OPF model, and, for that reason, it has linear and convex natures and can be solved by optimality in a reasonable time, however it cannot be assumed any level of accuracy, Ref [1, 2].

In the last years, optimization problems have begun to be handled by "modern heuristic" algorithms (heuristic, constructive heuristics,

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* Corresponding author:
E-mail addresses: phillipe.vilaca@urjc.es (P. Vilaça), street@ele.puc-rio.br (A. Street), josemanuel.colmenar@urjc.es (J.M. Colmenar).

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Nomenclature

Decision variables

α	Load flexibility in the AC-OPF model
$v,\ \theta$	Voltage magnitude and angle vectors
x	Investment state for transmission assets (binary)
$p,\ q$	Real and reactive generation vectors
$f^p,\ f^q$	Real and reactive power flow

Parameters:

$\rho^{if},\ \rho^{fi}$	Probability indexes used in the EBH algorithm
c	Nominal equipment investment cost
count_{MAX}	Stop criterion used in the EBH algorithm
r	Discount rate
$D^p,\ D^q$	Real and reactive demand
β	Penalization factor for *PNS*
$\underline{P},\overline{P}$	Bounds for real power
$\underline{Q},\overline{Q}$	Bounds for reactive power
$\underline{V},\overline{V}$	Voltage magnitude limit
$\underline{\theta},\overline{\theta}$	Voltage angle limit
N_{stop}	Stop criterion
np	Population size in EPSO algorithm
$G,\ B$	Conductance and susceptance
t	Index for stage
$\overline{S}$	Apparent power limit
$i,\ j$	Index for buses
PNS	Power not supplied
v	Velocity of a solution in the search space (EPSO)
$\underline{x},\ \overline{x}$	Lower and upper bounds for investment decisions
X	Set of solutions (EPSO)
Z^*	TEP Objective function
τ	Replication parameter (EPSO)

sets

$\mathscr{F}$	F&I boundary region
ϑ_x^1	Neighborhood 1 region
ϑ_x^1	Lower neighborhood region
$\mathscr{T}$	Set of stages in the planning horizon
$\mathscr{L}$	Set of pairs of buses defining new projects
$\mathscr{B}$	Set of all buses in the system
$\mathscr{B}_i$	Set of all buses connected to bus i
$\mathbb{B}$	Set of binary vectors with the dimension of the investment decisions
Ω_f	Region in the search space containing all feasible solutions (no-load shedding)
Ω_I	Region in the search space containing all infeasible solutions

metaheuristics, hyperheuristics) in which a candidate solution is tested against a constrained model, often in its full version. Even though the best solutions provided by that search mechanisms could be considered feasible, no proof regarding optimality can be drawn. On the other hand, approximated mathematical models that can be solved by optimality do not ensure the optimality in the original model, or worse, they do not ensure even feasibility in the original problem, and as a matter of truth, they are often infeasible [3].

In this sense, this work is motivated by combining solutions coming from modern heuristic algorithms with solutions coming from exact methods. On one hand, a suboptimal solution with ensured feasibility in the AC TEP model is taken from a metaheuristic algorithm and it is considered as an upper target regarding investments. On the other hand, an optimal solution from the DC TEP model is taken from a linear programming solver and it is considered as a lower target regarding investment. Our hypothesis relies on the fact that both solutions can be improved by an intelligent search mechanism that tries to reduce the gap between the lower and upper targets. Therefore, new reliable solutions with better quality can be obtained.

b Related work

To approach the related work, consider a generic non-linear and non-convex AC-TEP model which is represented by Problem 1 in Eq. (1) to Eq. (4), in which Eq. (1) is the objective function to be optimized (investment costs, operation costs, transmission losses, etc) Eq. (2). is related to equality constraints in the AC model, as the power balance equation, Eq. (3) is related to inequality constraints, as the apparent power in a branch and voltage variation in a bus, and Eq. (4) is related to the lower and upper limits of the decision variable, as output generators and investment in new assets.

Problem 1: Generic TEP Model

$$\min_x f(x) \tag{1}$$

Subject to:

$$g(x) = 0 \tag{2}$$

$$h(x) \le 0 \tag{3}$$

$$x \in \mathbb{X} \tag{4}$$

As this problem often cannot be solved by optimality in a reasonable time due to the curse of dimensionality, there are generally two ways to deal with the problem:

i Approximate/relax the model until it is possible to obtain an exact solution in a reasonable time.
ii Use the exact model and approximate the solution.

In the first case, the approximated model is represented by *Problem 2* described below by Eq. (5) to Eq. (8) which has equations with the same meanings of Problem 1, but now without the non-linearities and non-convexities, which means that an optimal solution can now be obtained by *Problem 2* (it is often claimed that the solution of *Problem 2* also represents a solution in *Problem 1*).

Problem 2: Generic approximated TEP Model

$$\min_x f'(x) \tag{5}$$

Subject to:

$$g'(x) = 0 \tag{6}$$

$$h'(x) \le 0 \tag{7}$$

$$x \in \mathbb{X} \tag{8}$$

Regarding the previously described approach *i*, TEP has been handled mainly by the approximated DC model in which the transmission losses, the reactive power, and the voltage bus variation are ignored in the model. The vast majority of TEP studies published in prestigious scientific journals use this model, as in Refs [4.–9]. In fact, almost all Transmission System Operators (TSOs) worldwide use this model in their analysis. However, in the last years, some works tried to find out how reliable this approximated model is when applied to a long-term analysis as TEP, as in Refs [10, 11]. Turns out the approximated DC model can deliver plans with severe infeasibilities in the real operation of power systems, mainly because in the real operation the transmission losses, reactive power, and voltage bus variation are quite sizeable.

Regarding now the second approach *ii*, which uses the original *Problem 1* – the AC TEP – and an approximated solution. This full model is considered an NP-hard problem, which means that the computational time required to solve the problem does not grow polynomially within the problem dimension, or in other words, the solution approach is not scalable. However, it is still possible to check how a given solution x performs under *Problem 1*. That is the main idea behind any modern heuristic algorithm, i.e; evaluate solutions x and change them following different mechanisms to improve their performance in *Problem 1*.

In the last years, some interesting works deal with TEP problems by using metaheuristic algorithms achieving good quality plans with a reasonable and justified computational cost, as in Ref [12–15]. However, and as mentioned before, these procedures lack mathematical proofs for optimality. Besides, the performance of the generated solutions can be very different when executing several runs of the optimization algorithm.

It is worth mentioning that nowadays some studies have been proposing tractable mathematical models for the AC-OPF problem using linearization, as in Ref. [16.–18], and convex relaxations based on semidefinite programming, as in Ref.[19]. These works truly represent a huge contribution to the scientific community regarding more precise models of the bulk systems, but as in the case of DC-OPF, these new approximated AC-OPF formulations also cannot ensure the feasibility in the original model. Finally, the main motivation in choosing the DC formulations as a lower target is the experience and dissemination of this model by the academy and power industry over the last twenty years.

c Contributions

Differently from previous work, in this paper, a combined approach is proposed where a global optimal solution from the DC TEP model and an approximated solution from the AC TEP model are used as input in a new MILP-Based Heuristic algorithm (MBH) that is responsible to deliver the final solution plan. The contributions of this work are threefold:

i. A thorough analysis of the performances of exact solutions from approximated TEP models and approximated solutions from full TEP models. The performances of these two approaches are related to the computational burden, investment costs, and infeasibilities regarding Power Not Supplied (PNS).
ii. A novel MILP-Based Heuristic algorithm that solves TEP problems using the optimal solution from the DC model and the approximated solution from the AC model. In this new algorithm, a movement function is proposed to reduce the gap between the lower target (exact DC solution) and the upper target (approximated AC solution).
iii. A mathematical proof of local optimality in the boundary between the feasible and infeasible region of the search space. This proof allows better exploitation in the search procedure, as well as an algorithmic gain in identifying more reliable solutions.
d. Structure of the paper, open science, and research integrity

This paper is organized as follows: Section 2 presents the mathematical formulation of the AC TEP full model and the approximated DC TEP model, Section 3 presents the proposed methodology with the feasible/infeasible boundary (called *F&I boundary*) characteristics. Section 4 brings a small tutorial system in which the solutions are analyzed, and the boundary is identified. The experimental experience is detailed in Section 5 in which two different instances are used to prove the effectiveness and robustness of the proposed method under both single-stage and multi-stage TEP approaches. Lastly, Section 6 brings the main conclusions about this work.

Finally, this work takes into account research integrity and open science practices for education and citizen in general. In this way, all codes produced in this paper, as well as the entire process through methodologies and results are available in the repositories in Ref [20, 21].

Mathematical formulation of the TEP problem

a Full AC TEP model

TEP is generally conducted considering the optimization of both investment and the security of supply. In this sense, the present work considers as an objective function the minimization of investment in new transmission assets over a planning horizon, besides the demand is considered inelastic, and penalization for Power Not Supplied (PNS) is also contemplated in the objective function. The AC TEP model is presented below by Eq. (9) to Eq. (20).

$$Z^* = \min_{\{x,\theta,v,f^p,\ f^q,\ p,q,\alpha\}} \sum_{t\in\mathscr{T}} \left[\sum_{(i,j)\in\mathscr{L}} \frac{x_{(i,j),t}\ .c_{(i,j)}}{(1+r)^t} + \sum_{i\in\mathscr{B}} \beta.(1-\alpha_{i,t}).D^p_{i,t} \right] \tag{9}$$

Subject to:

$$p_{i,t} + \sum_{j\in\mathscr{B}_i} f^p_{(i,j),t} - \alpha_{i,t} D^p_{i,t} = 0,\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{10}$$

$$q_{i,t} + \sum_{j\in\mathscr{B}_i} f^q_{(i,j),t} - \alpha_{i,t} D^q_{i,t} = 0,\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{11}$$

$$f^p_{(i,j),t} = x_{(i,j),t}\left[v^2_{i,t} G_{(i,j)} - v_{i,t} v_{j,t}\left(G_{(i,j)} cos\theta_{(i,j),t} + B_{(i,j)} sin\theta_{(i,j),t}\right)\right],\ \forall t\in\mathscr{T}, \forall(i,j)\in\mathscr{L} \tag{12}$$

$$f^q_{(i,j),t} = x_{(i,j),t}\left[-v^2_{i,t} B_{(i,j)} + v_{i,t} v_{j,t}\left(B_{(i,j)} cos\theta_{(i,j),t} - G_{(i,j)} sin\theta_{(i,j),t}\right)\right],\ \forall t\in\mathscr{T}, \forall(i,j)\in\mathscr{L} \tag{13}$$

$$\sqrt{\left(f^p_{(i,j),t}\right)^2 + \left(f^q_{(i,j),t}\right)^2} \le \overline{S}^2_{(i,j),t} =,\ \forall t\in\mathscr{T}, \forall(i,j)\in\mathscr{L} \tag{14}$$

$$0 \le \alpha_{i,t} \le 1,\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{15}$$

$$\underline{P}_{i,t} \le p_{i,t} \le \overline{P}_{i,t},\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{16}$$

$$\underline{Q}_{i,t} \le q_{i,t} \le \overline{Q}_{i,t},\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{17}$$

$$\underline{V}_{i,t} \le v_{i,t} \le \overline{V}_{i,t},\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{18}$$

$$\underline{\theta}_{i,t} \le \theta_{i,t} \le \overline{\theta}_{i,t},\ \forall t\in\mathscr{T},\ \forall i\in\mathscr{B} \tag{19}$$

$$x_{(i,j),t} \in \{0,1\},\ \forall t\in\mathscr{T}, \forall(i,j)\in\mathscr{L} \tag{20}$$

As mentioned before, the objective function in Eq. (9) is the minimization of the total system costs comprising investment and PNS costs, the decision variables are the investment state in new transmission assets x, the angle and magnitude voltages in the system nodes θ, v, the power output of generators p, q and the level of load shedding α in each node. The former provides flexibility enough to the AC-OPF to reduce the demand in node i at stage t $(D_{i,t})$ if this is required to maintain feasibility. The first Kirchhoff's law for active and reactive power balance in all system nodes is given by Eq. (10) and Eq. (11). The power flow represented by Eq. (12) and Eq. (13) is modelled as the product of binary variables x and the AC power flow equations, thus, for existing circuits x is fixed to 1, ensuring the power flow in that branch. However, for candidate circuits, x is binary which means that if a circuit is not selected to be installed on the grid, the power flow is forced to be zero. The thermal operation limits of the overhead transmission lines are considered in Eq. (14) while the AC flexibility variable limits are considered in Eq. (15). The lower and upper bounds for active and reactive generation, voltage magnitudes, and angles are considered in Eq. (16) to Eq. (19), respectively. For the sake of conciseness, in the above model, we use $\theta_{(i,j),t}$ to represent $\theta_{i,t} - \theta_{j,t}$. Therefore, the model should be read under this assumption. Finally, Eq. (20) presents the binary nature of investment decisions.

b The approximated DC TEP model

The DC TEP model is often based on the following approximations: branch resistances are negligible regarding the corresponding reactances ($G_{ij} \approx 0$), bus voltage magnitudes are near to the nominal value ($v_i = v_j = 1pu$) and voltage angle differences between nodes are small ($\theta_{ij} \approx 0 \rightarrow cos\theta_{ij} = 1, \ \ sin\theta_{ij} = \theta_{ij}$). For the sake of clarity, the resulting DC TEP model is presented by Eq. (21) to Eq. (29).

In this model, Eq. (22) represents the active power injection from bus i to j, Eq. (23) brings the active power injection, as this equation still contains a non-linear term the Big-M method is applied to avoid the presence of this term. Thus, if a candidate asset is selected, that is $x_{(i,j),t} = 1$, Eq. (24) gets forced to be an equality constraint. On the contrary, case $x_{(i,j),t} = 0$, the positive value of M ensures that Eq. (24) is not an active constraint. The active flows in the branches are limited at Eq. (25), while Eq. (26), Eq. (27), Eq. (28), and Eq. (29) have the same meaning of the corresponding equations in the AC-TEP model.

$$Z^* = \min_{\{x,\theta,v,f^p,\ p,\alpha\ \}} \sum_{t\in\mathscr{T}} \left[\sum_{(i,j)\in\mathscr{L}} \frac{x_{(i,j),t}\ .c_{(i,j)}}{(1+r)^t} + \sum_{i\in\mathscr{B}} \beta.(1-\alpha_{i,t}).PD_{i,t} \right] \tag{21}$$

Subject to:

$$p_{i,t} + \sum_{j\in\mathscr{B}_i} f^p_{(i,j),t} + \alpha_{i,t} D^p_{i,t} = 0, \ \forall t \in \mathscr{T}, \ \forall i \in \mathscr{B} \tag{22}$$

$$f^p_{(i,j),t} = -x_{(i,j),t} B_{(i,j)} \theta_{(i,j),t}, \ \forall t \in \mathscr{T}, \forall (i,j) \in \mathscr{L} \tag{23}$$

$$-M\left(1-x_{(i,j),t}\right) \le f^p_{(i,j),t} - x_{(i,j),t} B_{(i,j),t} \theta_{(i,j),t} \le M\left(1-x_{(i,j),t}\right), \ \forall t \in \mathscr{T}, \forall (i,j) \in \mathscr{L} \tag{24}$$

$$-\overline{F}_{(i,j),t} x_{(i,j),t} \le f^p_{(i,j),t} \le \overline{F}_{(i,j),t} x_{(i,j),t}, \ \forall t \in \mathscr{T}, \forall (i,j) \in \mathscr{L} \tag{25}$$

$$0 \le \alpha_{i,t} \le 1, \ \forall t \in \mathscr{T}, \ \forall i \in \mathscr{B} \tag{26}$$

$$\underline{P}_{i,t} \le p_{i,t} \le \overline{P}_{i,t}, \ \forall t \in \mathscr{T}, \ \forall i \in \mathscr{B} \tag{27}$$

$$\underline{\theta}_{i,t} \le \theta_{i,t} \le \overline{\theta}_{i,t}, \ \forall t \in \mathscr{T}, \ \forall i \in \mathscr{B} \tag{28}$$

$$x_{(i,j),t} \in \{0,1\}, \ \forall t \in \mathscr{T}, \forall (i,j) \in \mathscr{L} \tag{29}$$

Proposed model

In the present methodology, two solutions are considered as input:

i An approximated solution s_1 obtained from the AC TEP model from Eq. (9) to Eq. (20), a Mixed Integer Non-Linear Problem (MINLP). In this case, solved by employing a metaheuristic Algorithm.
ii An optimal exact solution s_2 obtained from the DC TEP model from Eq. (21) to Eq. (29), a Mixed Integer Linear Problem (MILP). In this case, employing the *Gurobi* solver 9.1.1 is used.

On the one hand, the approximate solution from the AC TEP model (s_1) does not have optimality proofs, although the feasibility is always ensured by the metaheuristic Algorithm, as will be observed in Section 3.d. On the other hand, the optimal solution from the DC TEP model (s_2) needs to be verified in the AC TEP model, some studies show that this solution may present several violations in the AC model, as in Ref [22, 23]. Thus, solution s_1 needs to be tested regarding the minimization of investments (while ensuring feasibility) and solution s_2 needs to be tested regarding the minimization of operation violations.

In the proposed model, the approximate AC TEP solution s_1 is considered an upper target, since it makes no sense to look for more expensive investment solutions than this feasible solution obtained by metaheuristic algorithm (with convergence guarantees). Conversely, the optimal DC solution s_2 is considered a lower target.

It is important to highlight that the upper and lower targets are not upper and lower bounds in the proposed methodology. In fact, it is impossible to assume any level of accuracy in the DC TEP model because it is an approximation of the AC TEP model. In this sense a relaxed TEP model (by convexification, for instance) could ensure this bound statement, however, the DC TEP model is, by far, the most widespread model by research academy and TSO worldwide, and for that reason, it was chosen in the proposed model.

Besides, the engineering knowledge obtained over several years in this field shows that the DC model undervalues transmission investments when compared to the AC model, Ref [22, 23]. Nevertheless, as will be present in section 3.b, the mentioned lower target statement does not compromise the methodology even when the AC TEP optimal solution is smaller than the DC TEP optimal solution.

a Moving solutions in the search space

The proposed MBH algorithm uses a movement function f that iteratively directs $s_1 \in \Omega_f$ through the lower target $s_2 \in \Omega_I$ (feasible to infeasible direction), and the $s_2 \in \Omega_I$ through the upper target $s_1 \in \Omega_f$ (infeasible to feasible direction). In the feasible to infeasible direction, an investment decision from $s_2 \in \Omega_I$ is randomly chosen and copied into $s_1 \in \Omega_f$. In this draw, the decision binary variables with zero value have a higher probability (ρ^{fi}) of being chosen since the objective is to reduce the investments in $s_1 \in \Omega_f$. In contrast, in the infeasible to feasible direction, an investment decision from $s_1 \in \Omega_f$ is randomly chosen and copied into $s_2 \in \Omega_I$. In this draw, the decision binary variables with a value of one have a higher probability (ρ^{if}) of being chosen since the objective is to reduce violations in $s_2 \in \Omega_I$ by considering new transmission assets.

At each iteration, the new solutions s_1 and s_2 are verified against the objective function presented in Eq. (9), if they have improved the targets are updated, that is when the new solution s_1 is less than the upper target and still feasible, then the upper target is updated by the new s_1. When the new solution s_2 presents fewer violations (PNS) than the lower target, then the lower target is updated by the new s_2. The process ends when none of the solutions are improved by a predefined number of iterations.

b. When solutions cross the feasibility boundary

During the iterations, the solution $s_1 \in \Omega_f$ is movement in the search space by coping the investment decisions of $s_2 \in \Omega_I$ with a higher probability to copy the binary investment decision variables equals zero. However, in this movement s_1 may become infeasible, which is not the goal as detailed before at the beginning of the section. In this case, the movement function f is applied in s_1 in direction to the upper target (always feasible), as in the infeasible to feasible direction detailed in Section 3.a. Nevertheless, before going back to the feasible region s_1 is compared with the lower target and, if it has fewer PNS, then the lower target is updated with s_1.

Similarly, as the solution $s_2 \in \Omega_I$ is movement in the search space by coping the investment decisions of $s_1 \in \Omega_f$ with a higher probability to copy the binary investment decision variables equals to one, s_2 may become feasible.

In this case, the movement function f is applied in s_2 in direction to

Algorithm 1
. Pseudocode of the EBH

1	**Input:** $\{s_1, p_1, i_1, s_2, p_2, i_2, \rho^{if}, \rho^{fi}, \text{count}_{MAX}\}$
2	**Initialization:** $\{s_{ut}, p_{ut}, i_{ut}\} \leftarrow \{s_1, p_1, i_1\}$, $\{s_{lt}, p_{lt}, i_{lt}\} \leftarrow \{s_2, p_2, i_2\}$, $\text{count}_1 = \text{count}_2 = 0$
3	**while** $\text{count}_1 < \text{count}_{MAX}$ or $\text{count}_2 < \text{count}_{MAX}$
4	**if** $p_1 = 0$
5	$\{s_1, p_1, i_1\} = f(s_1, s_{lt}, \rho^{fi})$
6	**else**
7	$\{s_1, p_1, i_1\} = f(s_1, s_{ut}, \rho^{if})$
8	**end**
9	**if** $p_2 \neq 0$
10	$\{s_2, p_2, i_2\} = f(s_2, s_{ut}, \rho^{if})$
11	**else**
12	$\{s_2, p_2, i_2\} = f(s_2, s_{lt}, \rho^{fi})$
13	**end**
14	**if** $(i_1 < i_{ut})$ *and* $p_1 = 0$
15	$\{s_{ut}, p_{ut}, i_{ut}\} \leftarrow \{s_1, p_1, i_1\}$
16	$\text{count}_1 = 0$
17	**else**
18	$\text{count}_1 + +$
19	**end**
20	**if** $(p_1 < p_{lt})$ *and* $p_1 \neq 0$
21	
21	$\{s_{lt}, p_{lt}, i_{lt}\} \leftarrow \{s_1, p_1, i_1\}$
22	$\text{count}_1 = 0$
23	**end**
24	**if** $(p_2 < p_{lt})$ *and* $p_2 \neq 0$
25	$\{s_{lt}, p_{lt}, i_{lt}\} \leftarrow \{s_2, p_2, i_2\}$
26	$\text{count}_2 = 0$
27	**else**
28	$\text{count}_2 + +$
29	**end**
30	**if** $(i_2 < i_{ut})$ *and* $p_2 = 0$
31	$\{s_{ut}, p_{ut}, i_{ut}\} \leftarrow \{s_2, p_2, i_2\}$
32	$\text{count}_2 = 0$
33	**end**
34	**end**
35	**Output:** $\{s_{ut}, p_{ut}, i_{ut}\}, \{s_{lt}, p_{lt}, i_{lt}\}$

the lower target (always infeasible), as in the feasible to infeasible direction detailed in Section 3.a. However, before going back to the infeasible region s_2 is compared with the upper target and, if it has less investment cost, then the upper target is updated with s_2. The EBH pseudocode is presented by Algorithm 1 below.

In the first line, the inputs are the parameter of the solution $s_1 \in \Omega_f$, that is, the solution, the penalization, and the investment cost (s_1, p_1, i_1), the parameter of solution $s_2 \in \Omega_I$, which are the solution, the penalization, and the investment cost (s_2, p_2, i_2), and the stopping criterion count_{MAX}.

Thus, in the second line, the upper and lower targets are initialized with the parameter of $s_1 \in \Omega_f$ and $s_2 \in \Omega_I$, respectively, and the counters count_1 and count_2 are initialized with zero, the EBH algorithm works trying to improve the solutions while at least one of these two counters are below count_{MAX}, as described in the third line. From lines 4 to 8, the movement function f is applied in s_1 as described in Section 3.a.

In the same way, this movement function is applied in s_2 from lines 9 to 13. Then, the new solution s_1 is compared against the upper target (in case s_1 is feasible) from line 14 to 19, and against the lower target (in case s_1 is infeasible) from lines 20 to 23.

The new solution s_2 is also compared against the lower target (in case s_2 is infeasible) from line 24 to 29, and against the upper target (in case s_2 is feasible) from lines 30 to 34. The new upper and lower targets are given in the last line, after EBH convergence. The behavior of the upper and lower target during the EBH procedure is presented in Fig. 1, while the overview diagram of the proposed approach is presented in Fig. 2.

The AC and DC TEP solutions are input in the model, when feasible s_1 changes iteratively in direction to s_{lt}, otherwise, it changes iteratively towards s_{ut}. On the other hand, when infeasible s_2 changes iteratively in direction to s_{ut}, if not it changes iteratively towards s_{lt}. Therefore, as s_{lt} is updated at each iteration to reduce the PNS levels (increasing the investment costs) and s_{ut} is updated at each iteration in order to reduce the investment costs (keeping null the PNS), the gap between these two solutions is reduced during this procedure. Finally, to complement the information of Algorithm 1, Fig. 2 presents an overview diagram of the proposed approach.

c. F&I boundary characteristics

The next paragraphs present the main characteristics of the F&I boundary, as well as its mathematical definition. Thus, for the sake of clarity, in the following definitions consider a solution only the binary decision vector representing the investment in new assets.

Definition 1. *Neighborhood 1* (ϑ_x^1)

Neighborhood 1 is the region close to solution x (binary vector representing investment decisions) in which the difference between any other solution y belonging to this region and x is related to only one asset status. This definition is expressed by Eq. (30) below, in this formulation $\mathbb{B}$ stands for the set of all binary vectors with dimension equal to the number of investment decisions.

$$\vartheta_x^1 = \{y \in \mathbb{B} \mid \ \| y - x \|_1 = 1\} \tag{30}$$

Definition 2. *Lower Neighborhood* ($\widehat{\vartheta_x^1}$)

The Lower Neighborhood of x is the subset of ϑ_x^1 with fewer investments Eq. (31). represents this set.

$$\widehat{\vartheta_x^1} = \left\{ y \in \vartheta_x^1 \| \ \| y \|_1 < \| \text{x} \|_1 \right\} \tag{31}$$

Definition 3. F&I boundary (F)

The F&I boundary is the set of feasible solutions whose Lower Neighborhoods are all infeasible points. So, if Ω_f is the set of all feasible solutions of $\mathscr{L}$, F can be defined as follows:

$$\mathscr{F} = \left\{ x \in \mathbb{B} \mid \widehat{\vartheta_x^1} \cap \Omega_f = \varnothing \right\} \tag{32}$$

Definition 4. Local optimality

A solution $x^* \in \Omega_f$ is local optimal if $\nexists y \in (\vartheta_{x^*}^1 \cap \Omega_f)$ such that $Z(y) < Z(x^*)$.

Theorem 1. All local optimal solutions, and consequently the global optimal solution, belong to the F&I boundary.

Proof: If $\vartheta_{x^*}^1 \cap \Omega_f = \varnothing$, x^* is local optimal by definition. If not, suppose $x^* \notin \mathscr{F}$. Then, according to **Definition 3**, there must exist at least one $y \in (\widehat{\vartheta_{x^*}^1} \cap \Omega_f)$. And, according to **Definition 2**, y should be equal to x except by one element equal to one in x that should value zero in y As all assets have a positive investment cost, $Z(y) < Z(x^*)$, contradicting the initial supposition that x^* is local optimal.

d. Further Comments on the new method

In short, the proposed methodology starts with a search procedure considering two good quality solutions: one approximated and feasible from the AC TEP with metaheuristic Algorithm, and another infeasible (hypothetically) from the DC TEP with a MILP solver. The MILP-based Heuristic algorithm is applied on both solutions and their movement naturally explores the F&I boundary, which is a promising region to be explored Fig. 3. represents these mentioned concepts in a two-dimension

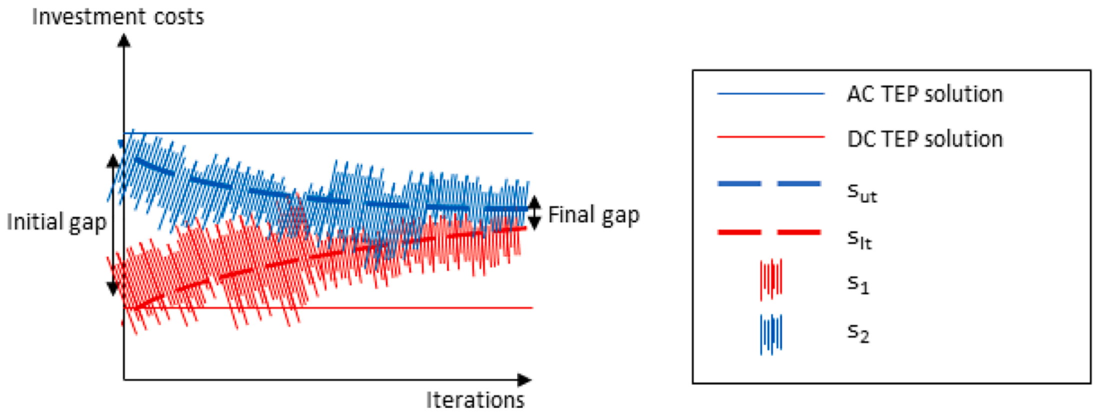

Fig. 1. Behavior of solutions during the process.

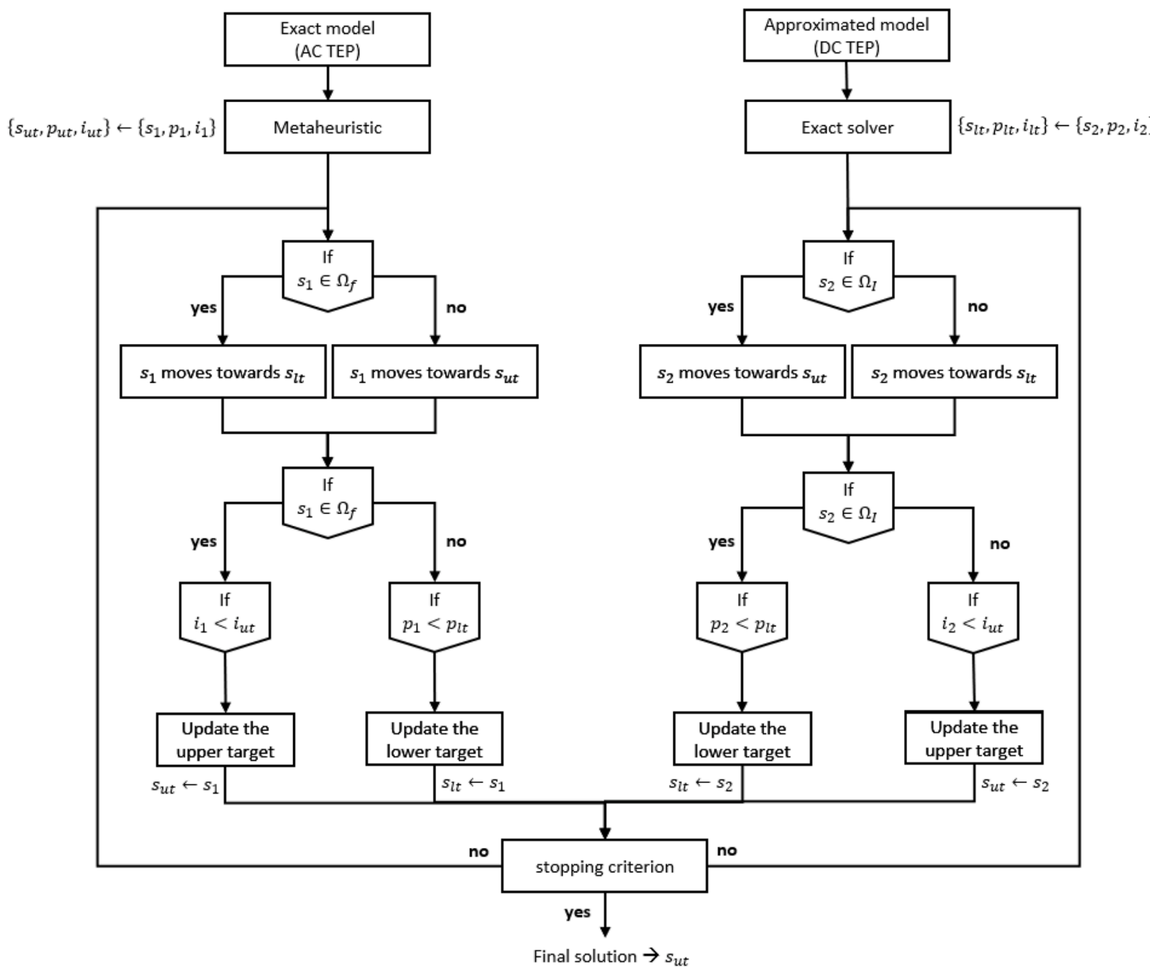

Fig. 2. Diagram overview of the proposed approach.

graph, just for the sake of didactics.

In this figure, the MBH algorithm receives the solutions s_1 and s_2, then it moves the solution s_1 towards the solution s_2, and solution s_2 towards the solution s_1. Both solutions are updated in the iterative process, exploring the boundary between the feasible and infeasible region, the F&I boundary. A hypothetical global optimal solution (red dot) is located at the F&I Boundary, a region in which the MBH algorithm naturally explores.

As can be noted, s_1 and s_2 oscillate between the feasible and the infeasible region of the AC TEP search space, the region that contains the optimal solution. As in the proposed model s_1 and s_2 are updated iteratively, the search is not restricted only to the path between the DC and AC TEP solution but to the entire F&I boundary. This is the justification for considering the approximated DC TEP model as a lower target. Notice that in Fig. 3, the optimal AC TEP solution is smaller than the optimal solution DC, considered as a lower target, but as s_1 and s_2 oscillate between the feasible and the infeasible region, the upper and lower targets will also be updated iteratively. In the next Section, a tutorial system will investigate the main definitions and ideas in this work.

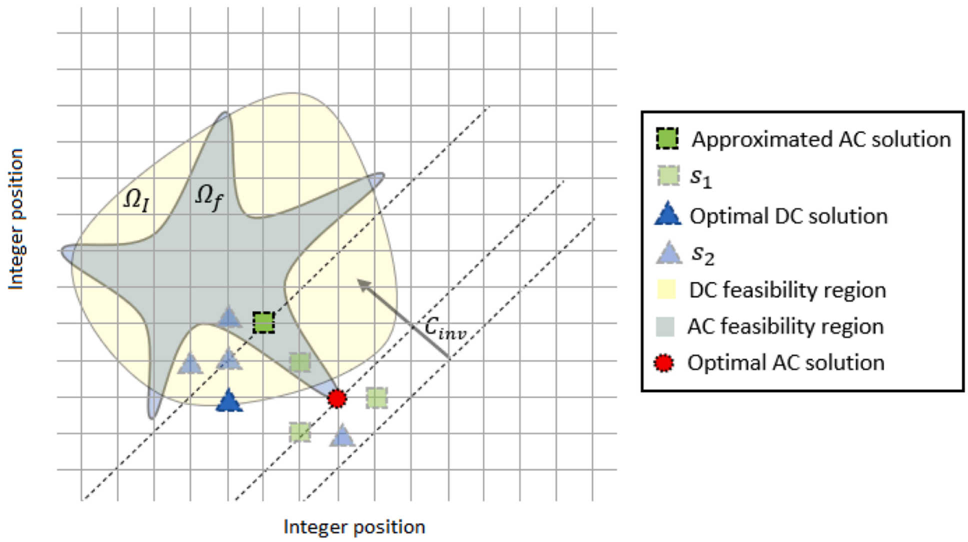

Figure 3. MBH algorithm: s_1 and s_2 exploring the F&I boundary.

e. Metaheuristic algorithm for the AC TEP model

In this work, the Evolutionary Particle Swarm Optimization (EPSO) is used to assess the AC TEP solution. It is important to highlight that any other metaheuristic algorithm could be utilized in the proposed approach. In our case, we have used EPSO in other past works and the algorithm often presents a better behavior than Genetic Algorithm and Particle Swarm Optimization, as in [10]. The main EPSO operators are described below, while its pseudocode is presented in Algorithm 2. The algorithm code, in *Matlab* language, is available in Ref [20].

First of all, EPSO inputs are the replication parameter (τ), the number of solutions (np), called as "individuals", in the set of solutions (X), called as "population", and the lower ($\underline{x}$) and upper ($\overline{x}$) bounds for investment decisions. Then, the population is randomly initialized, which is represented by the binary decision variables representing an investment in new assets in the AC TEP presented in Section 2.a. Then this population is evaluated using Eq. (9) and the interior point solver (for the AC dispatch considering the new pieces-of-equipment) available in Ref [15].

Following, the replication operator takes place by copying the population τ times, then the mutation operator changes the weights of movement rules for each of the cloned populations. In this way, new offsprings are created in each cloned population following the movement rule. Next, all individuals of each population are evaluated, and the selection block builds the new population, with np individuals, of the next generation until a pre-defined stopping criterion is reached.

Algorithm 2
: Pseudocode of the EPSO

1	**Input:** $\tau,\ np,\ \underline{x},\overline{x}$
2	**Initialization:** $W = rand(np,\tau) v = rand(\lvert x\rvert, \tau) X = round(rand(np, \lvert x\rvert)) Z(X)$, using Eq. (9)
3	**Replication:** $X_k = X,\ \forall k \in \{1,\dots,\tau\}$
4	**Mutation:** $W = 0.5 + rand - \frac{1}{e^W}$
5	**Reproduction:** $X_k = round(\ X_k + v^*\ W_{(k,1)}{}^* rand +\ W_{(k,2)}{}^* rand^* (x_k'' -\ X_k) +\ W_{(k,3)}{}^* rand^* (x_k' -\ X_k)$
6	**Evaluation:** $Z(\ X_k)$, using Eq. (9)
7 8	**Selection:** $X = \min(Z(\ X_k))$ **Stopping criterion:** N_{stop1} is a pre-defined number of iterations and N_{stop2} a pre-defined number of iterations without any improvement. ***if*** N_{stop1} or N_{stop2} is **True** *return*

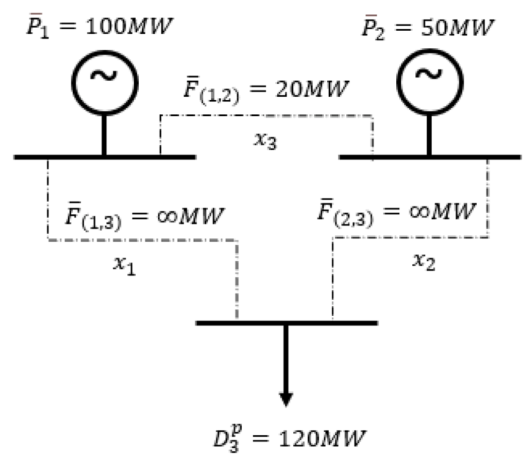

Figure 4. 3-bus tutorial system.

Tutorial system

In this Section, a tutorial system is studied to clarify the EBH algorithm and the F&I Boundary characteristics. The 3-bus tutorial system is presented in Fig. 4 and it has 2 generators with capacities $\overline{P_1} = 100MW$ and $\overline{P_2} = 50MW$, real power demand of $D_3^p = 120MW$, and the following branches limits: $\overline{F}_{(1,2)} = 20MW$, $\overline{F}_{(1,3)} = \infty MW$, and $\overline{F}_{(2,3)} = \infty MW$. The list of assets available to be installed on the grid is x_1, x_2 and x_3.

As this system is small and there is not a large list of possible solutions, for the sake of didactics and clarity all solutions are drawn as presented in Fig. 5. In this figure, the solution is a vector as $[x_1,\ x_2,\ x_3]'$.

Thus, in this tutorial system, the list of all solutions is $\mathscr{L} = \{A,\ B,\ C,\ D,\ E,\ F,\ G,\ H\}$ in which solution A represents no investment in new assets and solution H represents an investment in all assets of the list, the remaining intermediate solutions imply intermediates investments.

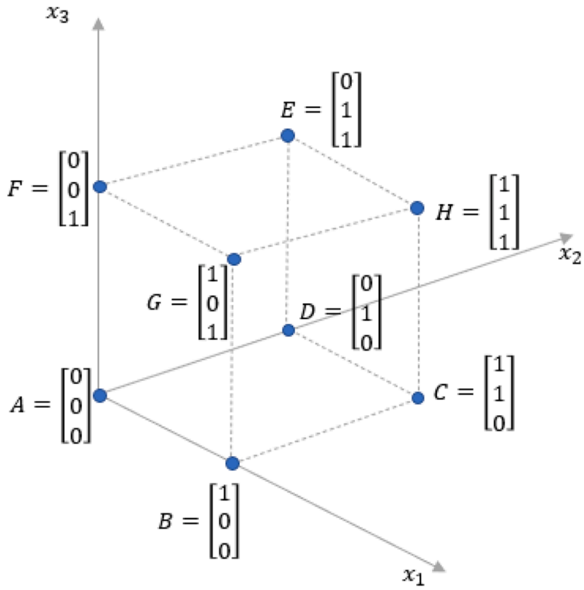

Figure 5. Complete list of solutions for the 3-bus tutorial system.

Comparing the solutions from Fig. 5 with the topology of the system presents in Fig. 4, solution A does not allow the two generators to meet the power demand, and therefore a PNS of 120 MW is necessary. Solution B represents the investment in the equipment x_1, which connects generator 1 to load 3. As the capacity of generator 1 is 100MW and the load in bus 3 is 120 M, a PNS of 20MW is obtained. Solution C indicates investment in the assets x_1 and x_2 connecting generators 1 and 2 to load 3 and, therefore, meeting the entire demand with no PNS. Solution D represents an investment in the asset x_2 which does not allow to meet the entire demand, this solution presents a PNS of 70 MW. Solution E symbolizes investment in x_2 and x_3, but again the solution is not enough to meet the entire demand and a PNS of 50 MW is taken. Solution F indicates that the only equipment to invest in is x_3, and therefore a PNS of 120 MW is defined in this solution. On the other hand, Solution G add the equipment x_1 and x_3 on the grid and, therefore, it can meet the demand with no PNS. Finally, solution H represents the investment in all of the three equipment, and it can meet the load with no PNS.

With the mentioned PNS per solution, the feasible set is $\Omega_f = \{C,\ G,\ H\}$ and the infeasible set is $\Omega_I = \{A,\ B,\ D,\ E,\ F\}$. An interesting fact about these solutions is that any modification related to a reduction in the investment in solutions G and C will result in an infeasible solution, or in other words, the Lower Neighborhood of G and C are $\widehat{\vartheta_G^1} = \{B, F\}$ and $\widehat{\vartheta_C^1} = \{B, D\}$, respectively, and they are infeasible. As the solutions G and C are feasible, they represent exactly the definition of F&I boundary in Eq. (32), that is, they are feasible solutions in which their Lower Neighborhoods are infeasible, therefore $\mathscr{T} = \{C, G\}$ Fig. 6. presents the set of solutions of the 3-bus tutorial system classified as infeasible and feasible ones.

Numerical simulations

a. Outline of the tests

In this section, two different experiments are presented: In the first one, a single-stage TEP model is conducted using the RTS-24 bus test system, and in the second experiment, a 10-years multi-stage TEP model is performed using the IEEE 118 bus test system. The RTS-24 bus test system's data is taken from Ref [24]., and the IEEE 118 bus test system's data is taken from Ref [25].

Regarding the AC TEP calculations, the interior point solver available in Ref [26]. is used to evaluate the solutions on the EPSO algorithm. We adapt the functions in this solver to consider the investment in new assets. Additionally, the objective function is also modified by considering a flexibility term to allow obtaining solutions in which the system is incapable of supplying all the demand, that is, in which nonzero Power not Supplied (PNS), or load-shedding, is required to regain feasibility. All *MATLAB* codes and results from the single-stage and multi-stage TEP are available in Ref [20]. and Ref [21]., respectively, as a way to ensure research integrity and open science practices for education and citizen in general Table 1. presents the parameters used in the EPSO algorithm and Table 2 presents simulation parameters used in both experiments, single-stage TEP and multi-stage TEP. These values

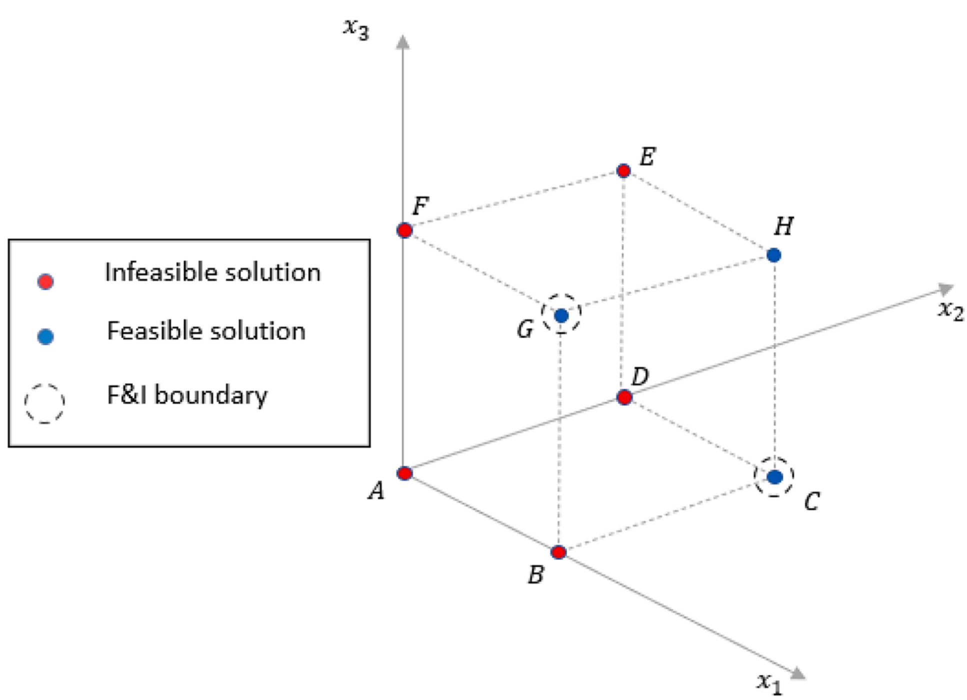

Figure 6. Classification of the list of solutions for the 3-bus tutorial system.

Table 1
Parameters used by the EPSO algorithm.

EPSO Parameters	Variable	Value
Max Iterations	N_{stop1}	100
Max Iterations without any improvement	N_{stop2}	30
Number of solutions	np	50
Replication parameter	r	3

Table 2
Parameters used in TEP simulations.

TEP Parameters	Variable	Value
Penalization factor for *PNS*	β	10^5
Discount rate (multi-stage TEP)	r	5%
Lower and upper voltage limit	$V_{i,p}^{MIN}$, $V_{i,p}^{MAX}$	0.95, 1.05 p. u
Probability parameter used in EBH	ρ^{if}, ρ^{fi}	10 and 5
Value of Lost Load	*VOLL*	5000$
Max additions per rights-of-way	n_{max}	3

were obtained after some preliminary experimentation (often called *tunning*), and it's intended to allow the replication of the proposed method.

b. Single-stage TEP

In this experiment, a single-stage TEP approach is conducted using the RTS 24 bus test system from Ref [24]. In this system, the generation capacity and the peak demand of the system are set at 10215 MW and 8550 MW. This test system has 24 buses distributed by two different voltage levels, one at 138 kV and another at 230 kV. This system has 33 generators, 38 branches connecting the nodes, and 41 pieces of equipment in the list of candidate equipment to be installed on the grid. The topology of the system is presented in Fig. 7.

First of all, AC TEP is conducted using the metaheuristic EPSO. As this solver can present different behaviors in solving the problem, the experiment is repeated 10 times, in this way, a thorough statistical analysis can be conducted to evaluate the performance of the proposed methodology Fig. 8. presents the behavior of the best solution per iteration over the 10 trials when using the EPSO algorithm to solve the TEP problem. As can be noticed, the stochastic behavior of the EPSO algorithm originates different solution plans for the RTS 24 bus.

Regarding the exact and optimal DC TEP solution, obtained using *Gurobi* Solver, it has an investment cost of 148 M$ and the list of equipment is presented in Table 3 below. Even though the DC TEP model is the most used mathematical model in TEP literature, the obtained solution in this experiment cannot ensure the proper attendance to the future load when tested against the full AC mathematical model, and for that reason, it presents a load shedding of 384.36 MW (4.5%).

Following, the MBH algorithm is applied using the final solution plan from EPSO (AC TEP model) and using the exact and optimal solution plan from the Gurobi solver (DC TEP model). In each of the 10 trials, MBH uses the final AC solution (no-load shedding) to improve the DC solution by reducing the degree of PNS by considering the investment costs of the AC solution (AC solution is considered as an upper target). Furthermore, MBH also uses the DC solution to improve the AC solution by reducing investment costs keeping no load shedding (DC solution is a lower target). The procedure updates both solutions iteratively, exploring naturally the F&I boundaries and reducing the investment gap between both solutions. The best feasible solution (lower investment with no load shedding) obtained in each trial by the MBH algorithm is presented in Fig. 9, while the best infeasible solution (lower PNS with a cost smaller than the current upper target) over the trials are presented in Fig. 10.

As stated before, the exploration of the F&I boundary allows the improvement of the upper target based on the lower target, and vice-versa. This iterative improvement culminates in a reduction in the investment gap between both solutions, as presented in Fig. 11 in which the initial gap between the DC and AC solution in each trial are compared against the gap of upper and lower targets after the MBH algorithm.

Nevertheless, the final solution plan that must be considered is the one without load shedding, once in the present mathematical formulation, the demand is considered as inelastic. Therefore, the MBH algorithm reduces the investment costs from the EPSO algorithm, even after the convergence of this metaheuristic, as can be seen in Fig. 12.

In this way, MBH allows an improvement in the final solution plan

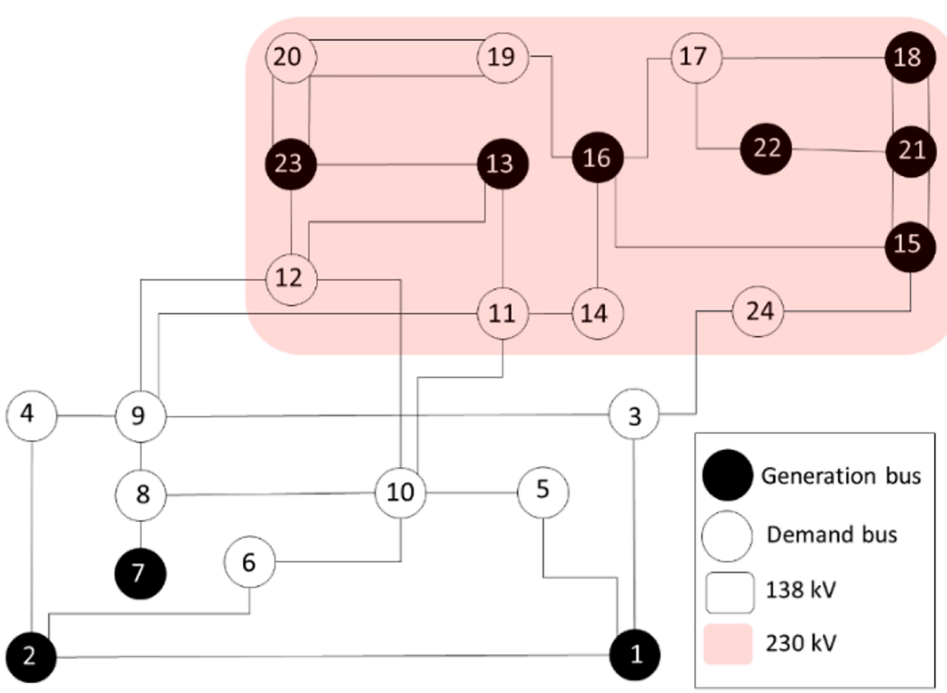

Figure 7. One-line diagram of RTS 24 bus (Single-Stage TEP).

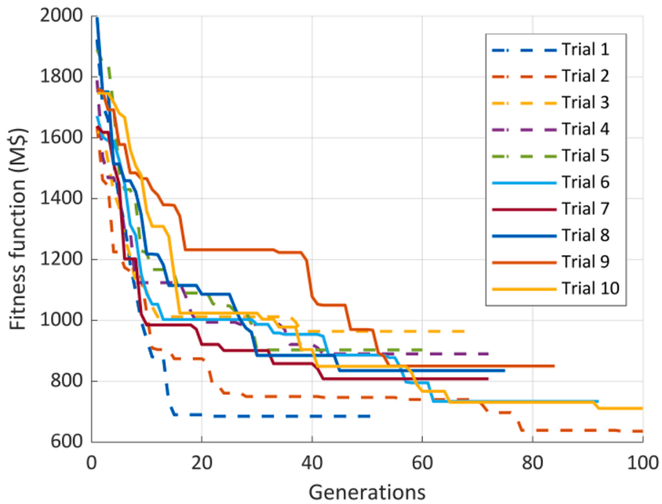

Figure 8. EPSO – Best solution per iteration over the 10 trials (Single-Stage TEP).

Table 3
Optimal solution plan using DC TEP formulation (Single-Stage TEP).

# equipment	Equipment description	Cost (M$)
1	Transformer connecting buses 3 to 24	50
2	138 kV line connecting buses 6 to 10	16
3	138 kV line connecting buses 7 to 8	16
4	138 kV line connecting buses 7 to 8	16
5	Transformer connecting buses 10 to 12	50

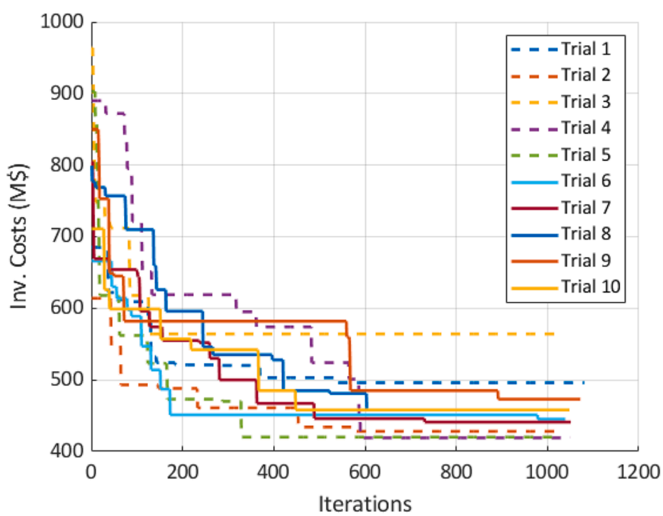

Figure 9. MBH – Best feasible solution per iteration over the 10 trials (Single-Stage TEP).

from the EPSO algorithm as can be noticed in Fig. 13 below. The average improvement obtained by using the MBH algorithm was about 42 %.

It is important to highlight that the EPSO algorithm calculates 150 (*np* x *r*) AC-OPFs at each generation, while MBH calculates 2 AC-OPFs (lower and upper targets) per iteration. Therefore, when analyzing the algorithm performances, it is not fair to compare the generations from EPSO in Fig. 8 against the iterations from MBH in Fig. 9. Instead, the behavior of the best solution (lower investment cost without PNS) against the number of AC-OPFs calculated should be chosen. Thus, Fig. 14 presents the behavior of EPSO and MBH for each trial over the AC-OPFs calculations.

Finally, and a last remark regarding the single-stage TEP simulations,

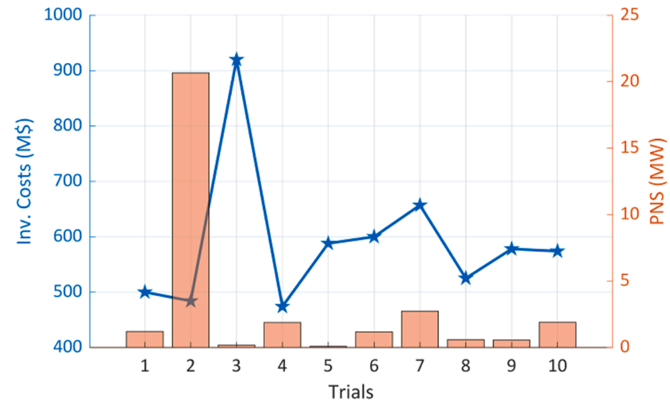

Figure 10. MBH – Best infeasible solution per iteration over the 10 trials (Single-Stage TEP).

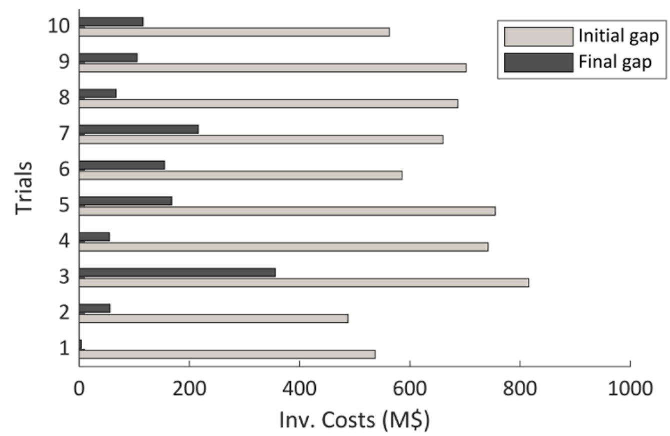

Figure 11. Gap between AC and DC solutions before and after MBH in each trial (Single-Stage TEP).

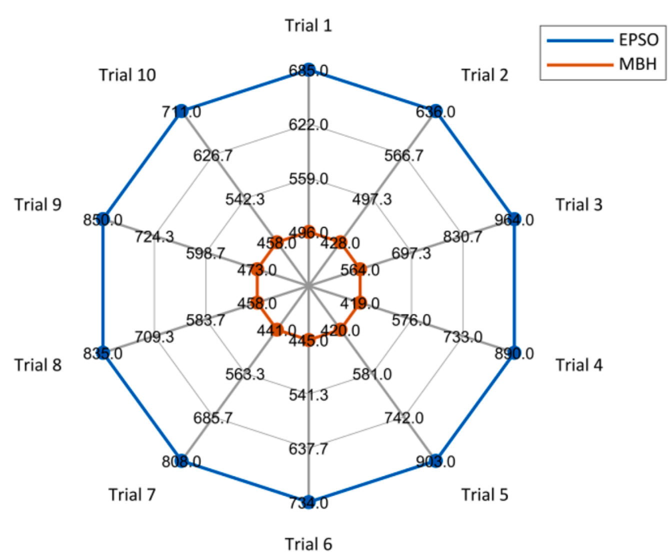

Figure 12. EPSO solutions against MBH solutions (Single-Stage TEP).

MBH was able to improve the solution for the RTS 24 bus presented in Ref [24]. with an investment cost of 515 M$ Table 4. presents the details of each piece of equipment selected for the final plan, which has an investment cost of 419 M$.

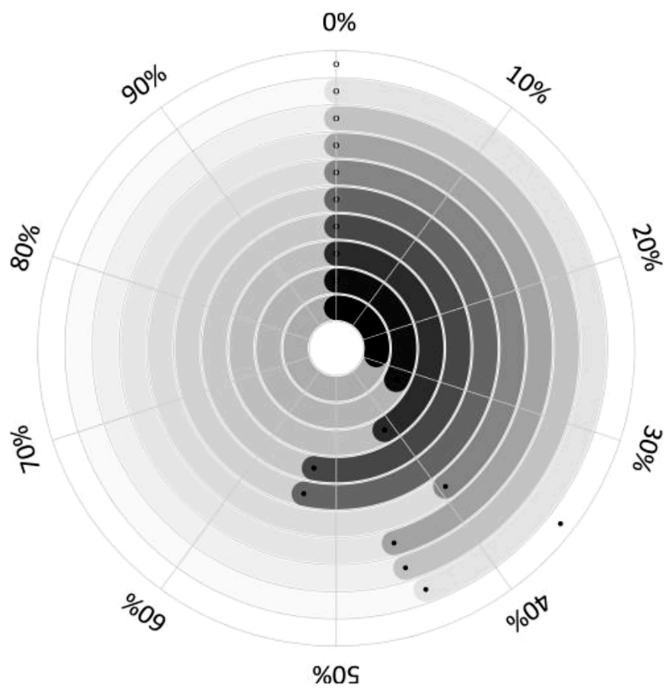

Figure 13. Improvements obtained by using the MBH algorithm in the Single-Stage TEP (each concentric circle represents a trial).

c. Multi-stage TEP

In this experiment, a 10-years multi-stage TEP approach is conducted using the IEEE 118 bus test system from Ref [25]. In this system, the generation capacity and the peak demand of the system for the last year of the planning horizon are 8270 MW and 6260 MW, respectively. This test system has 118 buses distributed by two different voltage levels, one at 138 kV and another at 345 kV. This system has 54 generators, 186 branches connecting the nodes, and 17 pieces of equipment in the list of candidate equipment to be installed on the grid. The topology of the system is presented in Fig. 15.

The demand is specified to grow at a rate of 3% per year, from the base year (4658MW) to the 10th year (6260 MW), the installation of new transmission assets is conducted yearly. As done in the single-stage experiment, AC TEP is conducted using the metaheuristic EPSO. The experiment is repeated 10 times, in this way, a better statistical analysis can be conducted to evaluate the performance of the proposed methodology Fig. 16. presents the behavior of the best solution per iteration over the 10 trials when using the EPSO algorithm to solve the TEP problem. As can be noticed, the stochastic behavior of the EPSO algorithm originates different solution plans for the IEEE 118 bus.

Regarding the exact and optimal DC TEP solution for the Multi-Stage TEP, obtained using *Gurobi* Solver, it has an investment cost of 17.4000 M$ and the list of equipment is presented in Table 5 below. This solution presents no load shedding when tested against the full AC TEP equations, however, it cannot be stated that this is the optimal AC solution, because the DC model is an approximated model and, for this reason, may not contain all the non-convex AC feasible region (see Fig. 3). This is the main reason why DC solutions cannot be considered lower bound in AC formulations.

In the proposed model the DC solution is handled as a lower target, as a way to push the AC solutions in its direction. Thus, the MBH algorithm is applied using the final solution plan from EPSO (AC TEP model) and using the exact and optimal solution plan from the Gurobi solver (DC TEP model). In this specific case, as the lower target has not any degree of infeasibility, MBH has used the DC solution to improve the AC solution by reducing investment costs keeping no load shedding (exploring naturally the F&I boundaries).

The best feasible solution (lower investment with no load shedding) obtained by the MBH algorithm was the same in all 10 trials and corresponds to postponing the equipment 1 presented in Table 6, to be installed in the 10th year, this solution has an investment cost of 15.8819 M$ Fig. 17. presents the initial gap between DC and AC solutions, against the final gap between lower and upper targets after MBH application. As can be noticed, the final gap is much less than the initial

Table 4
Best AC solution plan obtained by MBH algorithm for the RTS 24 bus (Single-Stage TEP).

# equipment	Equipment description	Cost (M$)
1	138 kV line connecting buses 2 to 4	33
2	Transformer connecting buses 3 to 24	50
3	Transformer connecting buses 3 to 24	50
4	138 kV line connecting buses 6 to 10	16
5	138 kV line connecting buses 7 to 8	16
6	138 kV line connecting buses 7 to 8	16
7	Transformer connecting buses 9 to 11	50
8	Transformer connecting buses 10 to 11	50
9	230 kV line connecting buses 11 to 13	66
10	230 kV line connecting buses 15 to 24	72

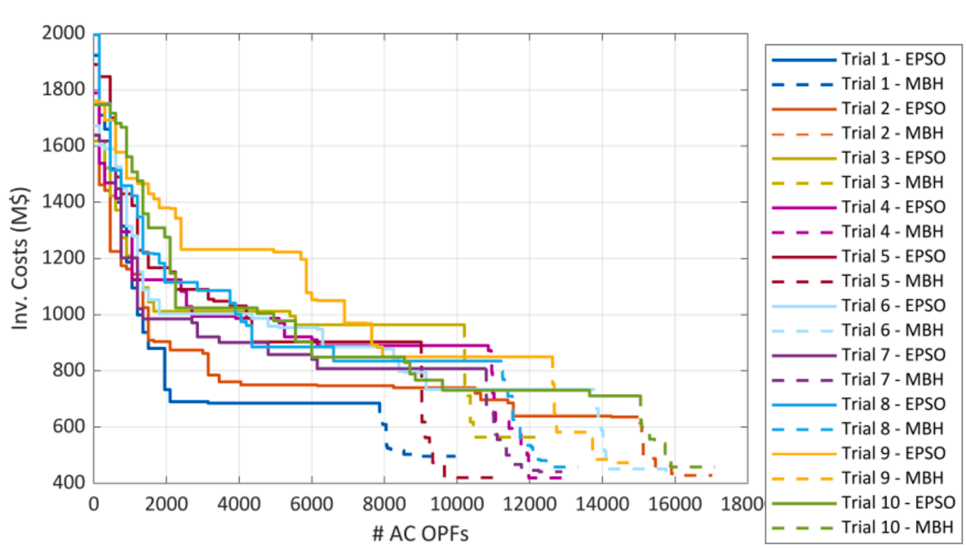

Figure 14. EPSO and MBH performances against the number of AC-OPFs calculated (Single-Stage TEP).

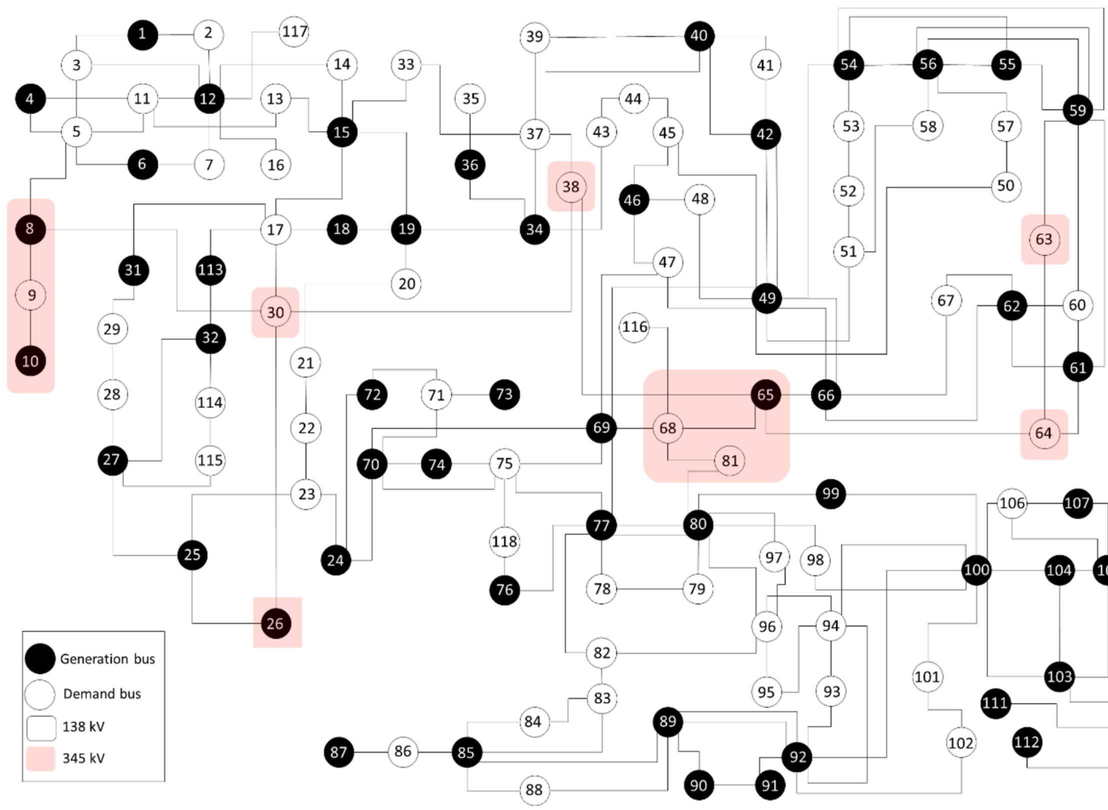

Figure 15. One-line diagram of IEEE 118 bus (Multi-stage TEP).

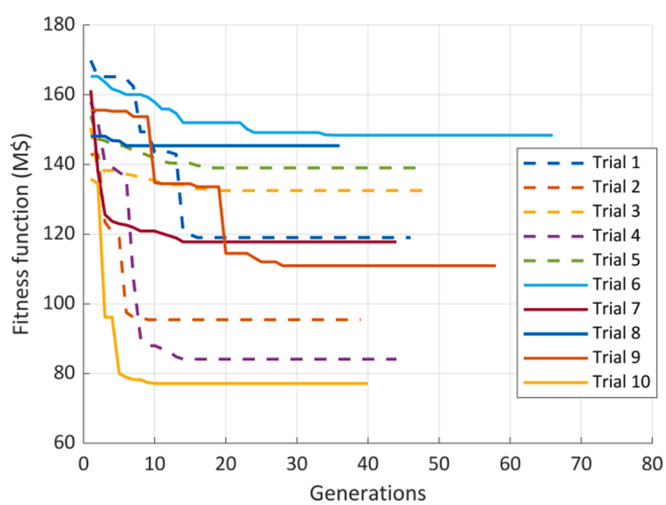

Figure 16. EPSO – Best solution per iteration over the 10 trials (Multi-stage TEP).

gap, this is because the MBH procedure can improve EPSO solution even when this metaheuristic presents a good behavior in finding the final solution, as shown in Fig. 18 in which the comparison of EPSO against MBH solutions is presented. Finally, the MBH improvement in the final solution plan from the EPSO algorithm is presented in Fig. 19 below. The average improvement obtained by using the MBH algorithm was about 85%.

In the same way, done in the Single-Stage TEP experiment, EPSO and MBH methods are compared using the best solution against the number of AC-OPFs calculated, which is presented in Fig. 20.

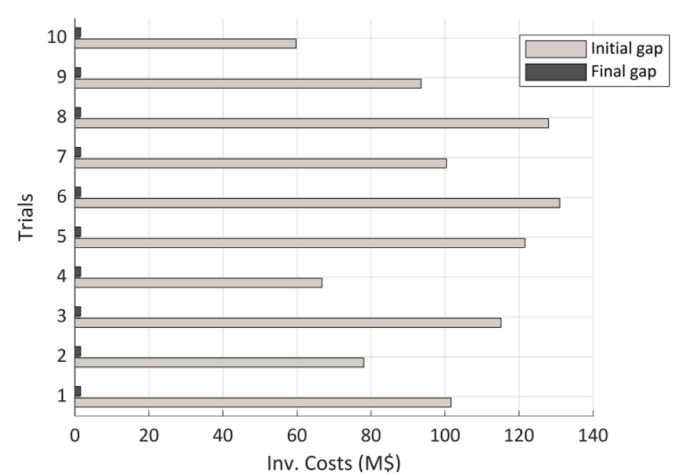

Figure 17. Gap between AC and DC solutions before and after MBH in each trial (Multi-Stage TEP).

Table 5
Optimal solution plan using DC TEP formulation (Multi-Stage TEP).

# equipment	Equipment description	Installation (year)	Present Cost (M$)
1	138 kV line connecting buses 77 to 78	5	7.0282
2	138 kV line connecting buses 3 to 5	10	5.3595
3	138 kV line connecting buses 8 to 5	10	5.0157

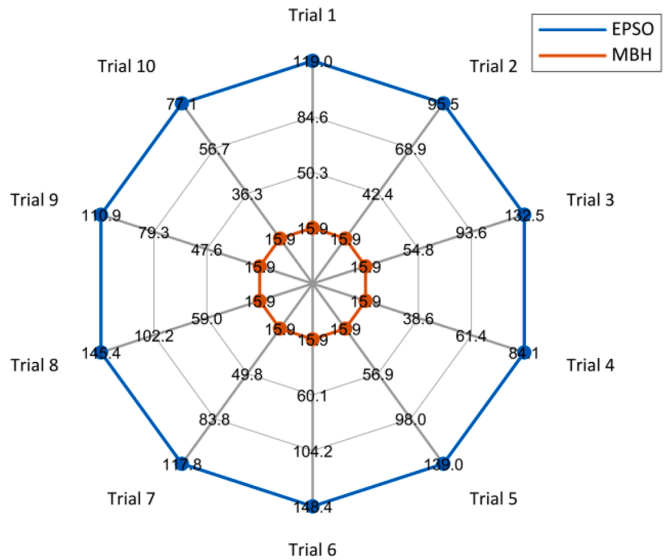

Figure 18. EPSO solutions against MBH solutions (Multi-Stage TEP).

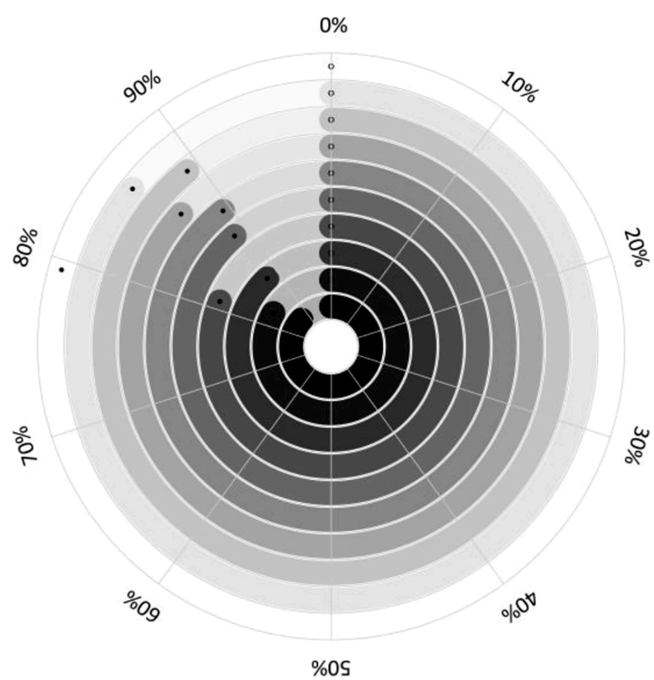

Figure 19. Improvements obtained by using the MBH algorithm in the Multi-Stage TEP (each concentric circle represents a trial).

Discussions

Regarding the experiments conducted with single-stage TEP and multi-stage TEP models, the results show that:

i Fig. 8 and Fig. 16 show that EPSO Algorithm presents a diverse behavior when solving the same problem under the same circumstances and premises. These figures corroborate to the existing skepticism in the application of several metaheuristics: On the one hand, the behavior is considerably unpredictable, and on the other hand, the solution obtained presents no aspect regarding its quality or proximity to optimality.

ii According to Table 3, the DC TEP solution can present a high level of infeasibility when it is applied in the AC TEP equations, even though the computational effort of the DC model (with a MILP solver) is much smaller than the AC model (with a modern heuristic). Therefore, this model is not a reliable approach to conduct any long-term analysis in a capital-intensive task.

iii According to the results presented in Fig. 11 and Fig. 17, solutions from non-convex and non-linear TEP models handled by metaheuristics and solutions from approximated and linearized TEP models handled by commercial MILP solvers can improve each other under a method, as proposed in this study, that allows accessing their position and the exploitation of interesting regions in the search space, as the F&I Boundary.

iv According to Table 6, the DC Multi-Stage TEP solution has presented no load shedding, similar to the AC Multi-Stage TEP solution. However, the former solution has presented a lower investment cost, which corroborates the initial hypothesis that the DC solution cannot be considered a lower bound in the AC formulation.

v According to Fig. 13 and Fig. 19, the proposed MBH has presented a remarkable performance by improving the final solution from a powerful metaheuristic. The proposed approach uses a smart procedure to explore a specific region in the search space, called F&I boundary, and the solution from the approximated DC model.

vi Regarding the Single-Stage TEP experiment, the proposed MBH algorithm was able to improve the solution for the RTS 24 bus (using the same planning conditions), presented in Ref [24]. with an investment cost of 515 M$. The MBH solution has an investment cost of 419 M$.

vii Regarding the Multi-Stage TEP experiment, the proposed MBH algorithm was able to improve the solution for the IEEE 118 bus (using the same planning conditions), presented in Ref [27]. (scenario D2) with an investment cost of 45.1 M$. The MBH plan counts with equipment totalizing 25.87 M$.

viii Based on the results presented in the last section, it can be concluded that the EPSO algorithm had an excellent performance in solving the Single-Stage TEP, but this statement should not be extended to the Multi-Stage TEP. Perhaps, for this case, EPSO should be associated with a local search to be greedier and exploit the search space in a more efficient way. However, the proposed MBH algorithm is independent of the metaheuristic choice. In fact, and according to the results presented in this work, even when the metaheuristic presents an excellent performance, MBH was able to improve the final solution, as in the Single-Stage TEP approach.

ix Finally, it can also be stated that the proposed MBH algorithm provides better or equal TEP solutions when it is applied using a DC solution (lower target) and an AC solution (upper target). In the case in which MBH cannot improve the final solution from metaheuristic, it can be stated that this solution is, at least, a sub-optimal solution for the problem, once the *F&I boundary* close to this sub-optimal solution doesn't present any other better plan.

Conclusions

In this paper, an Exact-Based Heuristic (EBH) algorithm is proposed to handle the Transmission Expansion Planning (TEP) problem in its two main formulations, the single-stage, and the multi-stage TEP models. In these formulations two types of solutions are admitted as input: an exact global optimal solution that is obtained using the approximated and widespread DC TEP model, and an approximated solution that is obtained using a Genetic Algorithm (GA) under an AC TEP model.

In the proposed method both solutions are improved based on their own position, that is, the AC solution is driven towards the DC solution and vice-versa. The EBH algorithm uses a movement function to perform the search in the F&I boundary. The output of the proposed method is two solutions, one feasible and the other with a small level of

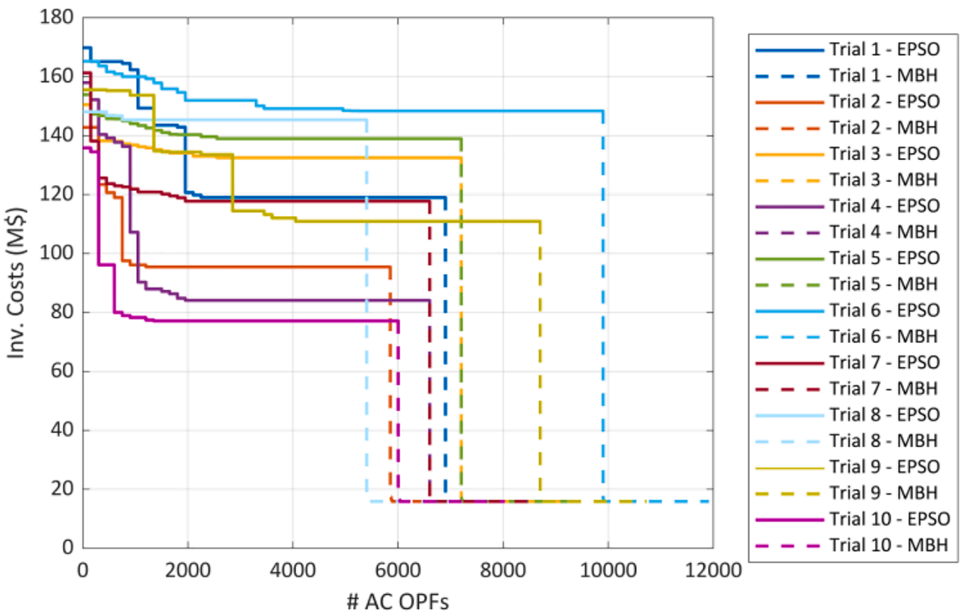

Figure 20. EPSO and MBH performances against the number of AC-OPFs calculated (Multi-Stage TEP).

infeasibilities regarding Power Not Supplied (PNS). The output solutions converge separately for the same region in the search space, reducing the GAP between their investment costs.

The efficacy and robustness of the proposed EBH algorithm were accessed considering two different test systems, the RTS 24-bus and the IEEE 118 bus. In this way, the proposed method was able to drastically improve the quality of the solutions coming from metaheuristics and MILP solvers. The remarkable results suggest that modern heuristics algorithms for full TEP models and exact-based solutions for approximated TEP models can be more reliable if they are compared and improved against each other, as in the proposed method.

CRediT author statement

- Phillipe Vilaça Gomes

Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing- Original draft preparation, Visualization.

- Alexandre Street

Conceptualization, Methodology, Validation, Formal analysis, Resources, Writing- Original draft preparation, Visualization.

- J. Manuel Colmenar

Conceptualization, Methodology, Validation, Formal analysis, Resources, Writing- Original draft preparation, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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