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Networks of random lasers: current perspective and future challenges [Invited]

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Abstract: Artificial neural networks are widely used in many different applications because of their ability to deal with a range of complex problems generally involving massive data sets. These networks are made up of nodes, connections, and nonlinear response connections, which are typically implemented as software code running on ordinary electronic computers. In such systems, electrons, with their advantages and drawbacks, are in charge of storing, processing, and transmitting information. Signal processing in the optical domain can provide ultrafast, parallel operation, nonlinear dynamics, and high energy efficiency, making photonics a suitable technology for the realization of neuroinspired computing platforms. This advantage stimulated the development of photonics neural networks based on single and multiple lasers with classical optical cavities. Recently, networks made of random lasers emerged as a novel concept that uses randomly placed scattering elements to create nonlinearity and complexity in photonics neural networks. In this review paper, we present the general framework for networks of coupled lasers, discuss recent advances in networks of random lasers, and outline future directions in this area. We also examine the challenges and limitations of using random lasers in photonic networks, as well as potential solutions. By harnessing the properties of random lasers, such as their unique spectral characteristics in pulsed emission mode and their robustness against noise, networks of interacting random lasers can explore new and exciting possibilities for photonics technology that could find applications in a variety of fields, including image recognition and encryption.

1. Introduction

The involvement of lasers in optoelectronic technology is deeply anchored in many aspects of our daily life and in countless research fields. Laser emission itself is a pivotal phenomenon on which to apply non-linear complex systems models. In fact, due to the nonlinear interaction of the light field with the active medium, spatiotemporal instabilities can give rise to chaotic behaviour that affect the spatial, spectral, and temporal emission properties [1].

For sake of clarity, chaotic dynamics is a deterministic phenomenon that arises from complex interactions within a nonlinear system [2,3]. It is distinct from random fluctuations that are produced by a stochastic process. However, not all nonlinear systems exhibit chaotic dynamics. In fact, the presence of nonlinearity is a necessary condition for chaos to arise, but it is not sufficient on its own.

Coherence, both in the spatial and temporal domain, is the most prominent characteristic of optical oscillators, i.e. lasers, that enables the phenomenon of synchronization of two or more resonators, exhibiting a rich variety of dynamical behaviour. Mutual interaction between different laser units is a crucial aspect in this phenomenon, as well as in interference and chaotic operation.

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Coupled lasers exhibit a rich set of properties related to synchronization, and the formation of large networks is a natural extension of this phenomenon, where complexity is gestated.

Random lasers (RLs) [4] have a set of qualities (both dynamical and spectral) that make them particularly well-suited in networks. Operating mostly in pulsed emission mode, they operate in two different emission regimes: a truly random regime, where each pulse is independent and exhibits unique spectral signatures, and a stable regime, where every shot is identical to the previous one. Control over this mode of operation can be obtained through mutual coupling of many RLs, thus giving rise to a rich phenomenological photonic network that could enable a wide range of applications exploiting the scalable nature of networked RLs. Typical temporal and spectral signatures of chaotic oscillations and randomly distributed frequency peaks of RLs are shown in Fig. 1(a) and 1(b), respectively.

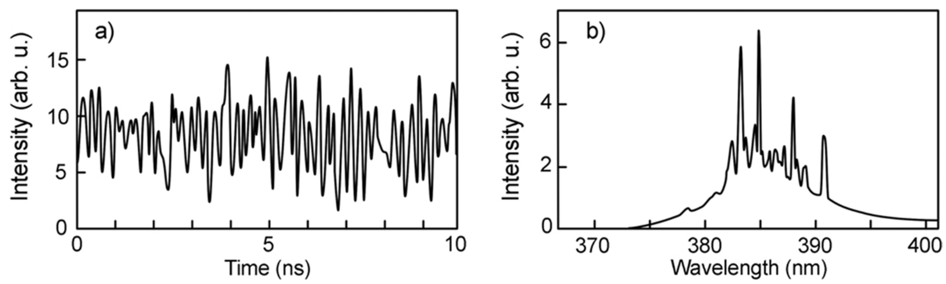

Fig. 1. Chaotic oscillations (a) at laser output produced by external feedback, adapted from [3], and (b) typical spectral signatures of RLs, adapted from [4].

The manuscript is organized as follows: in Section 2, we review systems of coupled lasers, describing chaotic dynamics and networks of lasers. In Section 3, neural networks and photonics neural networks based on conventional lasers are described. Finally, in Section 4, we introduce RLs with distributed and non-distributed feedback and review the state of the art of networks of coupled RLs.

2. Interacting lasers

2.1. Chaotic dynamics in lasers

In this paper we review the recent contributions that envision the realization of an optical network consisting of many coupled lasers. In this framework, the laser mutual and self-interactions must be controllable and affect the global emission properties. Grinding down the network scheme to its basic parts, we will proceed by discussing the interaction between a laser and its own reinjected field as well as the coupling between two different lasers. In the following we describe the nonlinear response to optical fields that can drive laser emission into the chaotic regime.

It has been proved in the '70s by Haken [5] that lasers are nonlinear systems similar to Lorenz model [6] and can show chaotic dynamics in their output powers. Lasers can be described by temporal differential equations with three nonlinearly coupled variables: field, polarization, and population inversion. If all three variables are included in the rate equations, the solutions for laser modes exhibit chaotic behaviour. However, if one variable relaxes much faster than the others, it could be adiabatically eliminated, and the system turns out to be non-chaotic. For instance, in solitary lasers, *i.e.*, emitters not subject to external perturbations, chaotic dynamics is usually absent.

From this viewpoint, lasers have been categorized into three classes, depending if their behaviour is dominated by a single equation for the field (class A, as He-Ne, Ar and dye lasers), or by two equations for the field and population inversion (class B, as Nd:YAG, CO_2 and semiconductor lasers) or by the full set of equations (class C, as NH_3 lasers) [7]. In class A or

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class B lasers, chaos cannot be observed unless the missing variables are introduced [8]. For example, class B lasers, in which only one additional variable is required, can be driven into chaos by an additional degree of freedom provided by an external element.

External optical feedback occurs when a laser interferes with its own delayed field. This condition is obtained by reinjecting a portion of the output beam back into the laser cavity once it had propagated through an external cavity, see Fig. 2(a). Alternatively, optoelectronic feedback can also produce chaos by proper modulation of the electrical pumping in semiconductor lasers. Depending on the delay introduced and the amount of re-injected light, lasers subject to external feedback show complex dynamical behaviours such as stable state, periodic, quasi-periodic oscillations and chaos [9–12]. From the historical perspective, the Lang-Kobayashi model [13] provides a theoretical framework for semiconductor lasers subject to external optical feedback, in which the round-trip delay introduced by the compound cavity drives the system.

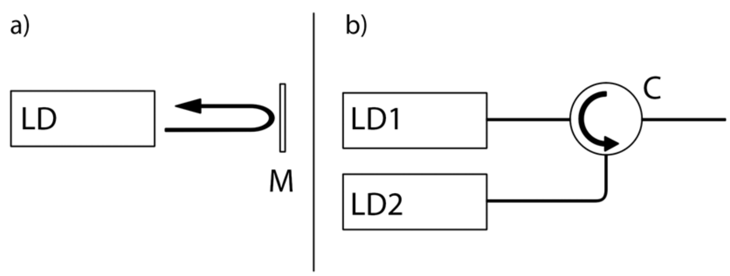

Fig. 2. Laser diode (LD) with external feedback (a) versus optical injection (b). In the schematics M is a mirror, LD1 the primary laser, LD2 the secondary laser and C the optical circulator.

External optical injection refers to the case in which the output beam of an additional laser is directed into the first laser cavity, as reported in Fig. 2(b). Semiconductor lasers, fibre lasers, and CO_2 lasers show unstable oscillations when subject to external optical injection or feedback [11]. In this case, not only the injected power but also the frequency detuning between external (primary) and injected (secondary) laser can be varied. Therefore, the operation regime in injected lasers can consist of stable locking, *i.e.*, when the secondary laser presents the same spectrum as the primary laser, or chaotic spectral emission [14,15].

To pursue stabilization by domesticating spatio-temporal chaos in single emitters, strategies that rely on delayed optical feedback have been successfully employed [16]. In particular, by engineering the interference of modes with random phases in disordered cavities with refractive index fluctuations, self-focusing instabilities have been prevented [17].

The ideas of chaos control [18] and chaos synchronization [19] stimulated the realization of electronic [20] and photonic [21] systems for chaos-based communication channels, in which two synchronized chaotic oscillators are used for encoding and decoding information on a secure channel [22]. In chaos-based optical communications, two semiconductor lasers (transmitter and receiver) are mutually coupled, and the message is superimposed as a small perturbation to the chaotic carrier. In this application, a small network of two lasers is specifically used for message encryption. Moreover, chaos in semiconductor lasers can find application in random bit generation, optical sensing and logical gates implementation [23].

2.2. Networks of lasers

Since lasers are inherently non-linear devices, they can be placed in a geometry that favours the mutual interaction in complex networks. For example, they can be arranged back-to-back in a regular chain or ring and interact with their neighbours by overlapping electric fields [24,25].

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Without instabilities, such network of N coherently coupled lasers will produce N^2 times as much power as a single one, giving rise to a synchronous steady emission of light [26].

However, semiconductor lasers do not tend to spontaneously lock the phase of single elements, even if they are coherently driven, therefore periodic pulsations, chaotic intensity fluctuations or even instability between adjacent elements can occur [27]. The breaking of coherent phase-locked regime was predicted for laser arrays with randomly spread frequency detuning between one another [28]. When the interaction is subject to a controllable delay, as in largely separated semiconductor lasers coupled through their propagating field by external feedback, the system can be driven into synchronized chaotic dynamics [29]. For a system composed of only two lasers, a delay between the dynamics of the leading laser and the lagging laser has been observed, fingerprint of spontaneous symmetry-breaking of the coupling [29].

In the case of a linear array of three single-mode Nd:YAG lasers interacting only via nearest-neighbour coupling, see schematics in Fig. 3, Terry *et al.* demonstrated that intensity synchronization occurs only between the two outer lasers, which show nearly identical intensity fluctuations and power spectra, while the central laser is not synchronized to the outer ones, although it mediates the interaction between the two surrounding elements [30].

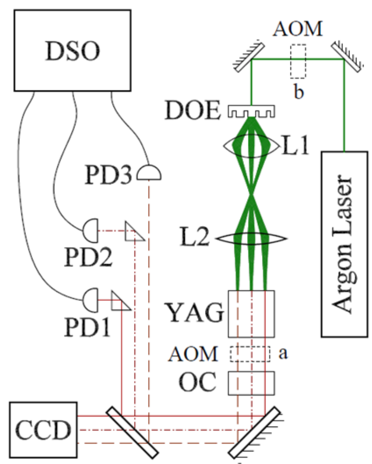

Fig. 3. Experimental system for generating three laterally coupled lasers in a single Nd:YAG crystal, adapted from [29]. The pump from an Ar laser is split in three beams by a diffractive optic element (DOE) and the beams are made parallel by the telescope made of lenses L1 and L2. An output coupler (OC) with 2% transmission conveys the far-field intensity pattern to a charge coupled device camara (CCD). The intensity of each beam is detected by three photodiodes (PD1, PD2 and PD3) and recorded by a digital sampling oscilloscope (DSO).

The intensity correlation of time series emission is a powerful method to estimate the degree of synchronization; however, it neglects the phase contribution of the laser fields. Measuring small angle interference of the laser beams allows to account for the synchronization of phase dynamics by evaluating the interference fringe visibility, that is directly proportional to the degree of mutual coherence [25,31]. Alternative methods based on fast Fourier transform algorithm have been also used to estimate the phase synchronization of interacting lasers [32].

Various configurations of two-dimensional laser arrays were studied by Oka *et al.* [33] by pumping a Nd:YAG active medium by spatially reconfigurable fibre coupled semiconductor lasers, as reported in Fig. 4. The authors demonstrate that phase locking of the pumping lasers array allow single mode output and scalable high emission power.

Synchronization obtained in networks of three and four chaotic semiconductor lasers can engender novel applications as encrypted communication channels based on fast random number

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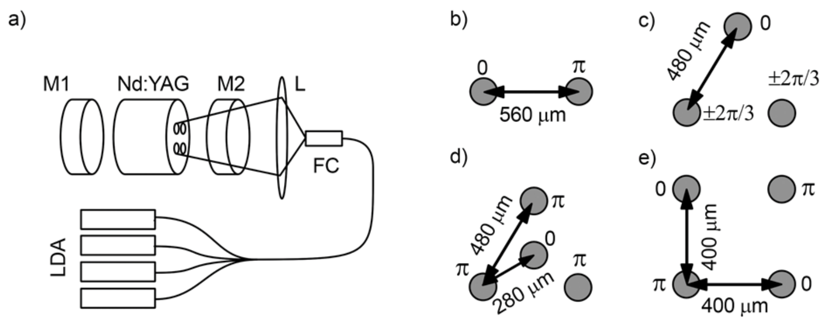

Fig. 4. Phase locking of an array of laser diodes [32]. (a) Experimental set-up: the laser cavity, consisting of two mirrors (M1, M2) and a Nd:YAG crystal, is pumped by a laser diode array (LDA) via fiber coupler (FC) and imaging lens (L). Different pumping configuration with two lasers in antiphase (b), three (c) and four lasers (d,e). Phase of each laser and distance of pumping beams in a plane perpendicular to the crystal are shown.

generators [34,35]. In fact, synchronization of networks of coupled chaotic lasers, separated by distances much greater than the individual laser coherence length, is the basis of fast (over Gbit/s rates), broadband and secure communication protocols in public channels, since they do not rely on deterministic algorithms for generating random numbers but on intrinsic chaotic laser dynamics [36].

Vertical cavity lasers (VCSELs) have been also used to realize photonic networks by using external diffraction coupling on a matrix of 8×8 emitters [37]. External feedback is provided by a reflective spatial light modulator (SLM) onto which the emission of the VCSEL array is imaged after passing through a diffractive optical element (DOE). Optical injection is provided by an external laser, see Fig. 5. An alternative approach relies on a single source which, with the use of a micromirror device and a spatial light modulator, can be turned into many nodes giving rise to a network up to 2025 nodes [38].

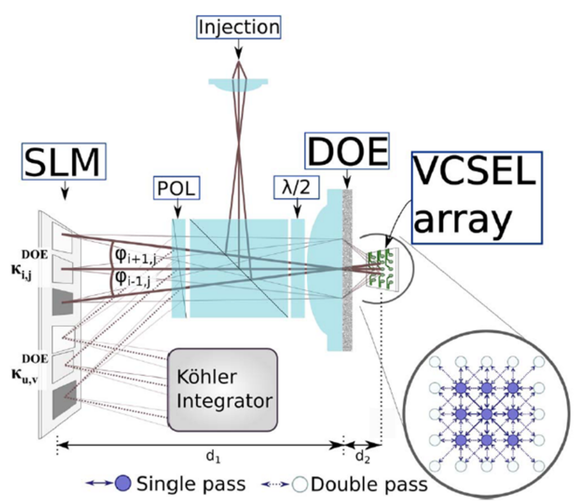

Fig. 5. Diffractive optical network by Brunner et al. [36]. The emitter is an array of 8×8 VCSELs subject to external optical feedback and injection, more details in the text.

The geometry and phase delay of thousands of coupled resonators can be controlled as proposed by Nixon *et al.* [39] and shown in Fig. 6. A degenerate Fabry-Perot (FP) cavity supports many

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independent lasers, since it contains a solid state active medium (Nd:YAG) and a spatial filtering mask that is projected on the cavity mirror plane by means of a two-lenses $4f$ telescope. In this setting, different network lattices can be created as a function of the mask geometry, in Fig. 6 (b) a Kagome lattice is shown as an example. The coupling between adjacent lasers in the lattice can be controlled by moving one cavity mirror, thus increasing the diffraction from each laser and the overlap between their fields. This interaction introduces losses, which depend on the relative phase between the considered lasers. In this way the authors could choose a negative coupling i.e., imposing a π phase difference between adjacent laser modes. When triangular of hexagonal lattices of laser networks are coherently pumped, a strong phase-locking is observed. Surprisingly, when a Kagome lattice is formed, by removing 1/4 of the elements from a triangular lattice such that any two triangular blocks share at most only one common site, the absence of long-range phase ordering in the laser network is found, as proven by a null fringe visibility in the far-field interference pattern.

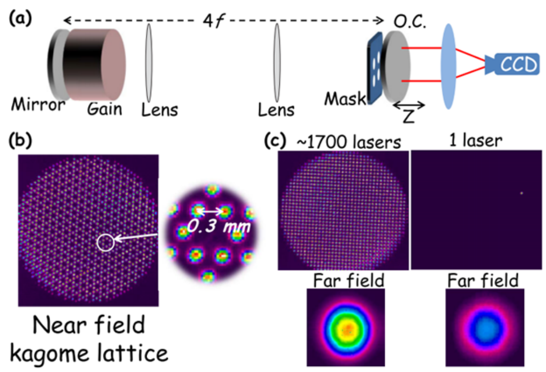

Fig. 6. Coupling more than a thousand independent lasers, reproduced with permission from [38]. (a) The experimental set-up consists of 4 f cavity formed by a mirror, two lenses and an output coupler (OC). The gain medium (Nd:YAG) is placed between the mirror and the first lens and the spatial filtering mask between the second lens and the OC. The beam profile exiting the OC is imaged onto a CCD. (b) Near-field intensity pattern of 1500 individual Gaussian laser beams arranged in 2D Kagome lattice. (c) Near-field (upper panel) and far-field (lower panel) intensity distribution of 1700 lasers and a single laser, respectively.

This is a geometric frustration phenomenon, which is typical of low-temperature magnets, and it manifests itself as an absence of long-range spin-like ordering in systems when competing interactions forbid the existence of a unique ground state [40,41]. This analogy with spin Hamiltonian system is helpful to understand the electromagnetic flow in nonlinear multimode photonic networks. For instance, optical phase transitions which resemble paramagnetic to ferromagnetic and spin-glass phase transitions occurring in spin networks have been reported depending on the network connectivity topology [42]. By further displacing the cavity mirror, next-nearest neighbour interaction can be introduced, and it allows to remove the frustration and restore long range phase ordering in the Kagome lattice laser network [39].

3. Complex networks

Networks of highly interconnected dynamical units are often addressed as *complex networks*. They are made of an ensemble of nodes connected by links that exhibit properties in between ordered and random lattices that can simulate the behaviour of many self-organizing physical systems [43,44]. From a topological point of view, the complexity of a network is estimated by graph theory and it roughly corresponds to the number of nodes and linkage patterns between them

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[45]. The dynamics of complex non-linear networks underlies a large amount of research fields spanning from biological systems and neuroscience to basic physics such as lasers, nanophotonics, fluid dynamics [46], but also encompasses economy, computer and social sciences [47,48].

In networks of mutually coupled oscillators, topological properties have a fundamental role in triggering cooperative phenomena such as synchronization [49]. In general, systems can exhibit external synchronization (between an oscillator and an external force) or mutual synchronization (between two nonlinear oscillators) [50]. A topological model of coupled oscillators with different connectivity was proposed to control the collective interactions between network elements and allows to reproduce self-organization and phase locking effects [51].

3.1. Neural networks

In the last few decades, a multidisciplinary effort has been carried out to create and optimize artificial neural networks, *i.e.*, complex computing systems that mimic the structural and functional features of the human brain [52]. In this framework artificial intelligence was raised, with the aim of approaching human-level capabilities and performance in tasks that are challenging for traditional computer architectures due to their sequential and digital operation.

The significant advantages of a neural network over a conventional processor are parallelism (data processed simultaneously, as opposed to sequentially) and bringing together both data storage and computing. Thus, machine learning algorithms based on neural networks can process information on hardware that naturally supports parallel operation and adapt for training rather than being explicitly designed by a programmer [53]. For developing a simplified architecture of neuromorphic computing, a large numbers of nonlinear elements (neurons) have to be connected (synaptic links) [54]. Each artificial neuron, after receiving inputs from other neurons, engenders a nonlinear response that can be broadcast to other neurons, possibly including itself, as reported in Fig. 7. Three types of design of artificial network architecture have been proposed [53]below: a) feed-forward neural network, in which signals travel through successive layers of neurons and are usually employed in deep neural networks; b) recurrent neural network, in which each neuron can receive outputs from previous but also from subsequent neurons, thus lacking a defied direction of the information flow; c) convolutional neural networks, in which layers are grouped into so-called nodes that successively process information to detect hierarchical patterns in data.

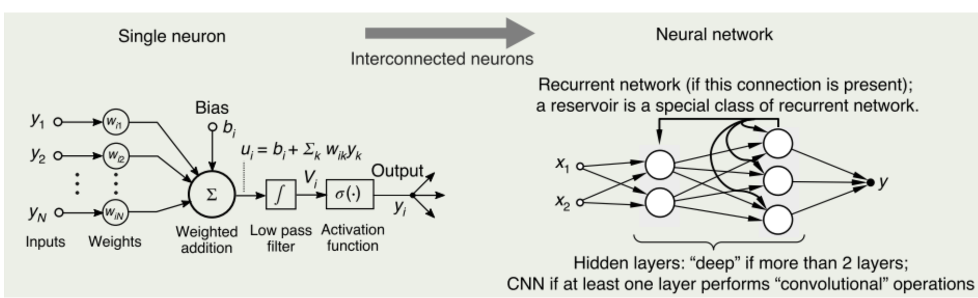

Fig. 7. Schematics of an artificial neuron and a neural network architecture, reproduced from [52].

Applications of artificial neural networks in machine learning can be found in improving speed and efficiency of computations, unmatched by software implementations; recognizing image contents (e.g. human faces, handwritten digits or even reporting on input images) as well as time varying signals like recorded speech, to name a few [54,55].

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3.2. Photonic neural networks

Massively parallel interconnections between elements are key requirement for optimizing neuromorphic computing. However, implementing electronic hardware that employ metal wiring for connections is not practical, since they have to rely on shared digital communication bus, imposing a trade-off at bandwidth interconnectivity [56]. Moreover, due to the interactions between electrons, parallel information-transmission along the same channel would be limited. On the other hand, the optical approach does not impose any trade-off and photons have negligible interaction between them, therefore making it suitable for parallel data analysis and fast machine learning directly on-chip [52]. Photonic neural networks allow to encode information in the optical field (power, phase, polarization, mode, wavelength) and have been tackled in multiple fashions using photonic technologies [57].

Nevertheless, photonic neurons have to accomplish some basic requirements, such as being able to integrate multiple optical inputs, use controllable nonlinearities and produce an optical output as response used to drive other elements [53].

It is important to stress that the goal of neuromorphic photonic processors is not to replace conventional computers, but to enable applications that are unreachable at present by conventional technology, such as those that require low latency, high bandwidth, and low energies [58,59]. Photonic networks designed as neuromorphic system have been used to perform unsupervised pattern recognition directly in the optical domain, thus pushing photonic networks as a ground-breaking technology in artificial intelligence and deep-learning applications [60].

By employing a single laser diode (LD) with controlled external feedback, an all-optical neural network was trained by large input–output mappings and stochastic learning algorithm [61]. In this approach, the longitudinal modes of the LD are spatially separated after passing a pair of gratings, and a liquid crystal display (LCD) operating in transmission is used to assign specific weights to each mode. Longitudinal modes behave as neurons since their power depends nonlinearly on the induced optical feedback, which is controlled via a pixel mask imposed to the LCD, thus moulding the emission spectrum during the learning process, see Fig. 8.

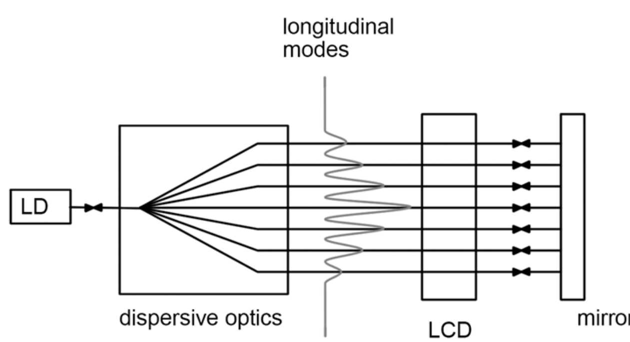

Fig. 8. Optical neural networks based on FP laser diode (LD) subject to controlled external feedback, adapted from [60].

Moreover, all optical logic gates have been realized in similar LD subject to external optical feedback by weighting the longitudinal modes [62].

Recently, a different approach for machine learning has gained momentum: reservoir computing [63,64]. This scheme is alternative to the deep learning strategy since it trains only a few output connections. It aims at processing and learning only a single output layer, while a reservoir made up of a large and fixed network of connections with random weights is left untrained. The reservoir generates many complex behaviours in response to input perturbations, then the read-out layer is trained to interpret the network state and provide it as the output, see Fig. 9.

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The reservoir is requested to exhibit nonlinearities, furnish a stable output if the input remains unchanged and give similar responses for slightly different inputs, so that the learning procedure can lead to a satisfactory generalization. Many types of physical substrates can be used as a reservoir, including a variety of photonic systems [54,65,66]. Remarkably, a laser subject to external feedback can be used as the only nonlinear node of a neural network for performing complex tasks, as speech recognition based on reservoir computing [67]. In this case the reservoir, rather than being comprised of spatially arranged neurons, is composed of temporally distributed pulses in a fibre resulting from time-delayed reinjection of the LD emission. An advantage of using injection and feedback lasers for building optical neural networks is that they manage to speed up the processing time of complex tasks [68]. Spatial modes of an injection locked VCSEL have been used as network nodes with variable weights changed by an external mask [69].

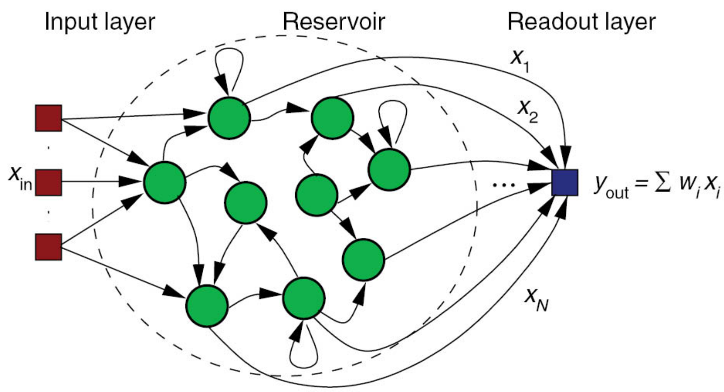

Fig. 9. Standard layout of a reservoir computer, comprising an input layer (red), the reservoir (green) with randomized but fixed connections, and the linear readout layer (blue) [66].

For instance, information processing using a nonlinear optoelectronic oscillator subjected to delayed feedback allowed pattern recognition when the complex dynamical system was trained by a large input data stream [70]. The capabilities of reservoir computers can be further enhanced by including delayed variables able to induce complex spatiotemporally mixed networks [67,71]. This architecture allows to reduce the number of elements by employing fewer nonlinear nodes with delayed feedback.

Finally, photonic reservoir computing has been proved to give rise to scalable and spatially extended networks and achieve dynamical complexity by combining optical delays with standard optical elements such as amplifiers, detectors, modulators, interferometers or nonlinearities in semiconductor lasers [37,53].

4. Networks of random lasers

4.1. Random lasers

The first idea of a laser with a cavity based on diffusive elements was proposed in the late 1960s [72], with a device consisting of a gain medium placed between a mirror and a scattering surface. Lasing action and emission spectra showing randomly distributed narrow peaks in frequency have been observed years later from optically pumped colloidal solutions of dye and scatters [73] and semiconductor powders [74]. These devices were dubbed RLs [75], recalling the random distribution in space, size, and shape of the scattering particles, which provide optical feedback for lasing action. The spectral signature of RLs is an unpredictable and non-periodic distribution of lasing modes in frequency, which depends on the scattering properties of the cavity. It can result in a multi-mode emission spectrum with variable degree of spatial coherence [76].

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Since their inception, RLs have been realized in many configurations and with many materials [77]. Depending on the organization of the scattering elements relative to the gain material in a RL, two types of devices can be realized [78]: *i)* RLs with distributed feedback, in which the scattering elements are embedded and randomly distributed inside the active medium; *ii)* RLs with non-distributed feedback, *i.e.* with scatters located at the edges of the active material and enclosing it [78,79], see Fig. 10.

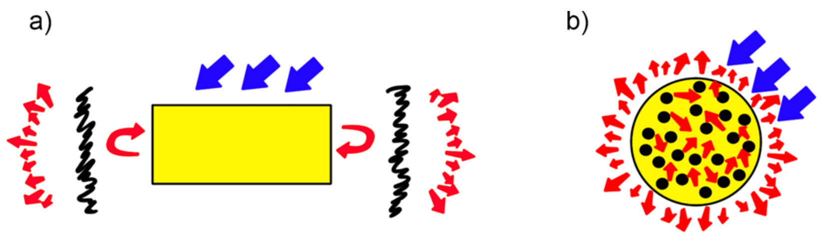

Fig. 10. Schematic description of non-distributed (a) and distributed (b) feedback in random lasers, adapted from [78]: gain medium (yellow area), optical pump (blue arrows), scatters (black areas) and lasing emission (red arrows).

4.2. Distributed feedback

In RLs with distributed feedback, the spatial distribution of scatters and the transverse profile of the pump on the active medium define the actual resonator. Therefore, a modulation of the pump beam profile can selectively activate certain nodes and branches, resulting in different emission spectra and output beam profiles, ultimately allowing to control the laser emission [80–83]. Since specific spectral signatures can be obtained by proper modulations of the optical pump profile, this kind of RLs found applications in counterfeiting [84].

In RLs with distributed feedback, scattering particles placed inside the active medium, represent the nodes of a complex network, linked to each other by the pumped regions, as proposed by Hofner *et al.* [85]. The authors reported on an optically pumped RL consisting of multiple quantum wells in which holes at random positions were etched during the epitaxial growth. Each hole is a scattering element regarded as a network node. The observation of a time varying emission spectrum during the pump pulse is attributed to a dynamical behaviour of the network in which connections between scatters vary their phase contribution.

A relevant connection between RLs and disordered networks is the lasing network (LANER) concept proposed by Lepri *et al.* [86] shown in Fig. 11. The authors induced random lasing action in complex networks with optical gain by tailoring the connectivity between elements to realize topological disordered states. Single mode erbium doped fibres, which produce optical amplification under continuous wave pumping, were connected by couplers that introduce disorder in the topological scheme (see Fig. 11). The emission spectrum observed at the network end resembles the statistics of a random laser and reflects the topological disorder induced by the connections. Therefore, the entire network can be considered as a laser in a disordered cavity *i.e.*, a discrete random laser with controllable complexity [87].

The proposed laser network shows close analogies with the quantum graph theory of chaos [88]. Theoretically it was described by a propagation matrix formalism containing the connections (topology) and the lengths and gains (metrics). A global scattering matrix, containing the transfer properties of the optical couplers connecting the links, provides the suitable boundary conditions for the fields. The probability density function of the intensity of lasing modes showed Lévy distribution with heavy-tails fluctuations. In addition, a formalism that models such network taking into account the nonlinearities of optical amplification strongly connects graphs theory to RLs [89]

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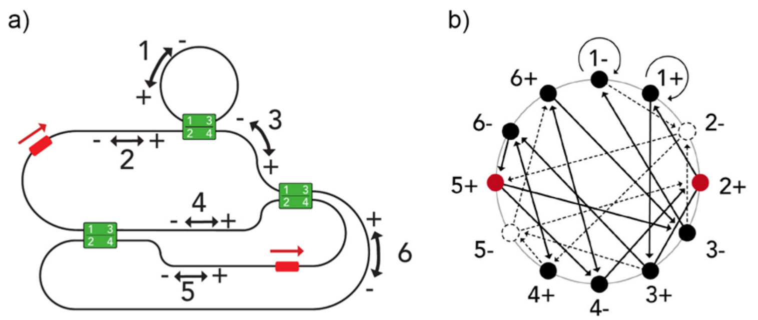

Fig. 11. Laser network (LANER) concept [86]: (a) schematic representation of the set-up, consisting of fiber segments (black cords), gain sections (red blocks), isolators (red arrows) and 2 × 2 splitters (green blocks). (b) Graph representation of the LANER, fiber segments are represented by nodes and splitters correspond to connections. Black and red nodes refer to passive and active fiber segments, respectively.

Another example of a RL network is the one realized by Gaio *et al.*, exploiting multiple light scattering localization in a network of subwavelength waveguides, see Fig. 12 [90]. The RLs network was made of complex meshes of polymeric active nanofibers in which the nodes induced multiple scattering. By exploiting the interconnection of many active waveguides, a network of up to 200 nodes was achieved. The behaviour of the global optical network is accurately described by a graph solution of Maxwell's equations that highlight the central role of network topology and connectivity in promoting and selecting random lasing modes.

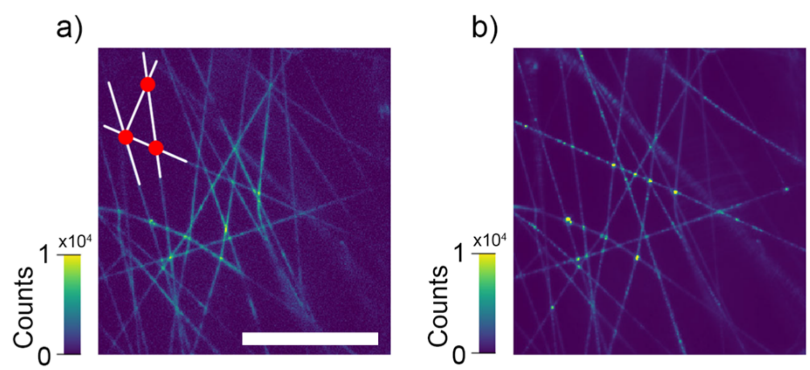

Fig. 12. Nanophotonic RL on a graph [90]. Far-field images of fluorescence (a) and lasing (b) from a network of free-standing subwavelength electrospun polymer nanofibres embedded with dye. Scale bar is 100 μm.

Following the approach of RL networks in polymer fibres, as in Ref. [90], fibres bundles were fabricated that give rise to localized RLs in micrometric domains and show high spatial correlations over distant nodes [91]. Such heterogeneous RLs were made of ribbon-like and highly porous fibres that can be alternatively switched on and off by tuning the pumping intensity. This photonic network of RLs allowed the observation of the replica symmetry breaking phenomenon by imaging the near-field emission of the network and then quantifying the shot-to-shot intensity fluctuation at the single pixel level.

RLs with distributed feedback have also been demonstrated as basic building blocks for security key distribution in a decentralized network [92]. In [92], authors proposed a fibre-based RL consisting of a dye solution embedded with nematic liquid crystals, enclosed into a hollow core

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fibre and placed between the two cleaved ends of two optical fibres. One fibre is used to heat the active medium by photothermal effect with an infrared source and the other for monitoring the output spectrum. Heating of the RL changes the phase state of the liquid crystals and, when an external voltage is applied, output power shows a hysteresis cycle as a function of the intensity of the infrared source. The resulting bistability and the randomness of the emitted spectrum are used to demonstrate a decentralized encrypted network in which the nodes are the proposed RLs.

4.3. Non-distributed feedback

In RLs with non-distributed feedback, the cavity consists of an active material enclosed between scattering surfaces or volumes, similarly to the lasers operating in a resonant FP cavity, with the difference that disordered material constitute the mirrors, hence introducing a wavelength-dependent phase and amplitude modulation that selects the resonant modes [79].

Recently, by employing a series of interconnected RLs with non-distributed feedback, novel networks have been proposed [93,94]. In these RLs networks, uncorrelated disordered lasers can be synchronized through adaptive pumping, thus laying a solid foundation for controlling laser

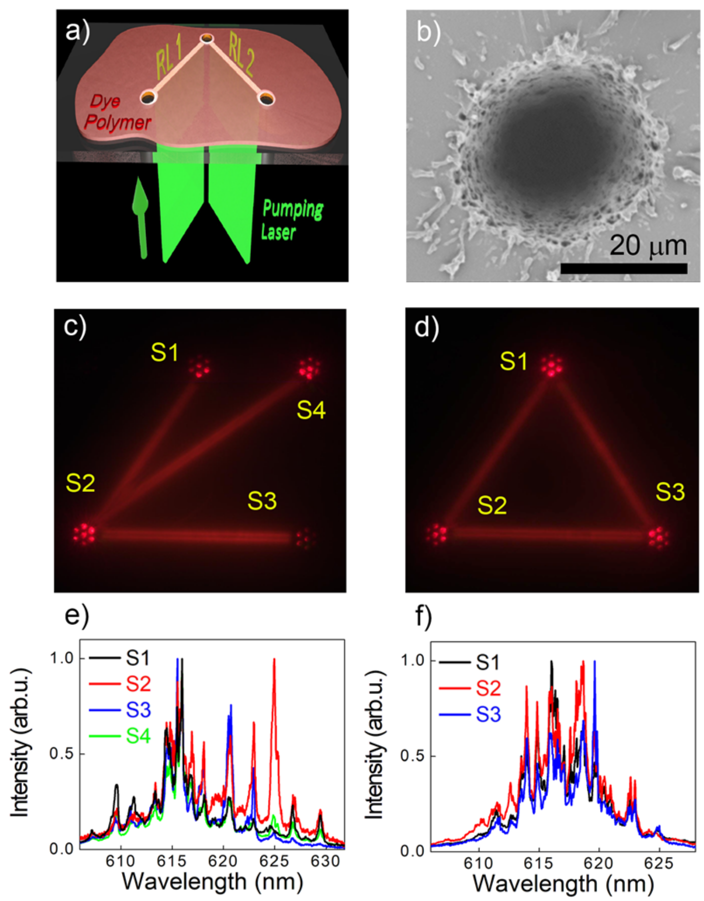

Fig. 13. Network of coupled RLs with non-distributed feedback [93]. Figurative representation of the set-up (a), detail of a scanning electronic microscope image of a scattering center (b), open (c) and closed (d) networks of three resonators and their emission spectra, (e) and (f) respectively, from each scattering center (S1, S2, S3, S4).

emission in disordered environments as well as photonic interconnection and complex systems, see Fig. 13.

The mutual coupling between two and three individual RLs was engineered and tested in a thin biopolymer platform in which the network interaction was determined by the spectral rearrangement of the compound emission compared to individual RLs [93]. In this system scattering regions were created by femtosecond laser ablation of the polymer matrix and the optical gain was induced along narrow lines connecting the scattering centres. A single RL is independent on the rest of the network if only one line is optically pumped between the two scattering centres that act as feedback elements for that resonator [79] and as coupling elements and output couplers for the entire network. Networks with different topologies have been tested, as for example with three and four scattering centres which have been connected to form networks of three RLs, as shown in Fig. 13 c and d. Strongly correlated multi-mode spectra, as shown in Fig. 13 e and f, have been attributed to the mutual coupling between adjacent RLs. The emission stability, in terms of peak spectral distribution, over 2000 successive pumping shots, was proved to be remarkably long [93].

Networks of coupled RLs have been also obtained with fibre based RLs in which non-distributed feedback is provided by rough internal walls of hollow core optical fibres filled with dye solution [94]. In that work, simple networks of two and three nodes have been demonstrated, where each node consists of a fibre RL and connections are made of standard optical fibre. By varying the pump area and energy of each laser, a specific node can be set as the primary laser and injects the secondary nodes. Highly correlated spectra have been obtained through injection locking, as shown in Fig. 14. Making use of the single shot synchronized output, spectral coding was proposed as an efficient way to share information between nodes. In this way, wavelength coding in coupled RL networks exhibiting multimode spectral emission, emerged as a reliable way to communicate between distant nodes and it could be a basis of networking in complex optical systems.

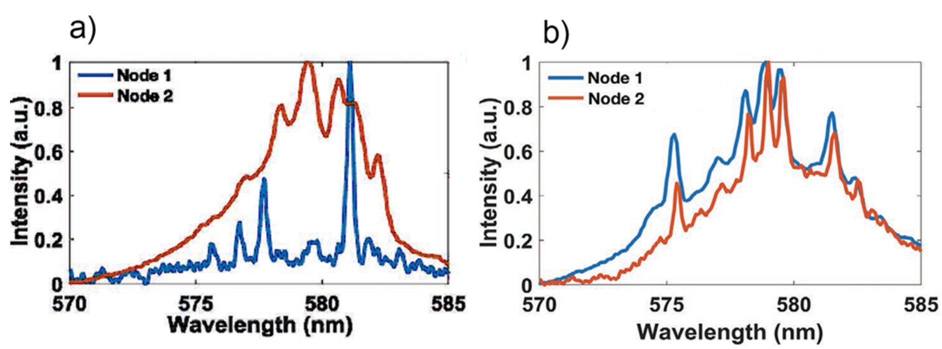

Fig. 14. Fibre based networks of coupled RLs [94]. Emission spectra from two unconnected (a) and connected (b) RLs.

5. Outlook

RLs as a single network of connected scattering centres and as nodes of networks of coupled resonators show the complexity, non-linearity and fabrication flexibility that are required by efficient photonics platforms dedicated to neural networks implementation. The relevance of these devices is given by the general advantage of signal processing in the optical domain and by the specific emission properties of RLs, as for example randomly distributed lasing frequencies which can be used for mapping the network inputs and outputs. The randomness of the lasers can provide robustness against noise and improve performance. Another potential advantage

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is the RL unique spectral signatures, which depends on the coupling between elements in the network. This feature can be used for encoding and transmitting information in applications such as encryption and to produce unclonable devices.

In this manuscript, we reviewed these devices in the general context of optical oscillators subject to external perturbation, such as optical injection, external feedback and mutual coupling. Rich temporal dynamical behaviour, including chaos, self-pulsation and complex signal generation, not extensively studied in RLs, is expected in these devices, adding more degrees of freedom to their use in networks.

From a practical point of view, current challenges and requirements are controlling the pumping and modulation, using materials capable to sustain continuous lasing emission (to realize time-varying signal reservoir computing) and that show high nonlinearities; finding novel designs and architectures to lower the intensity requested to operate. One of the main challenges is controlling the emission modes of the lasers. Another challenge is the getting rid of noise and instability in the network, which could affect the reliability and accuracy of the system.

Further to those related to materials, the challenges and limitations can be addressed through the development of advanced control mechanisms and network architectures, as well as through careful design and testing of the system. Overcoming these hurdles will mean that coding data to feed RL neural networks and setting weights that direct the computing will be facilitated and thus, photonic processing will be eased. We believe that the properties of RL networks will trigger the study of more complex architectures, possibly paving the way to realize novel photonic neural networks.

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