CERTIFICATE

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the Advisory Committee

Controlling the bursting size in the two-dimensional Rulkov model

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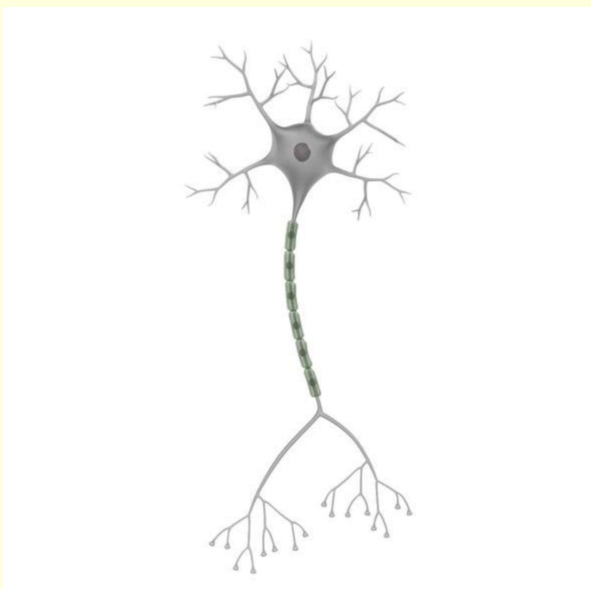
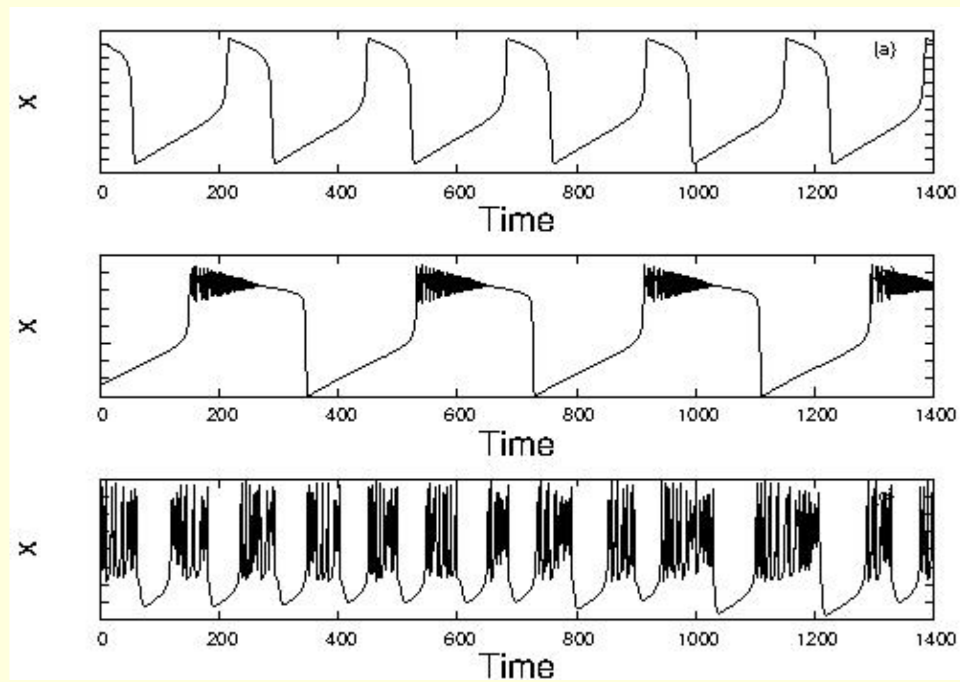
Dept. of Physics, Universidad Rey Juan Carlos, Madrid, Spain

The Rulkov map

$$x_{n+1} = \frac{\alpha}{(1 + x_n^2)} + y_n$$

$$y_{n+1} = y_n - \sigma x_n - \beta,$$





The Rulkov map

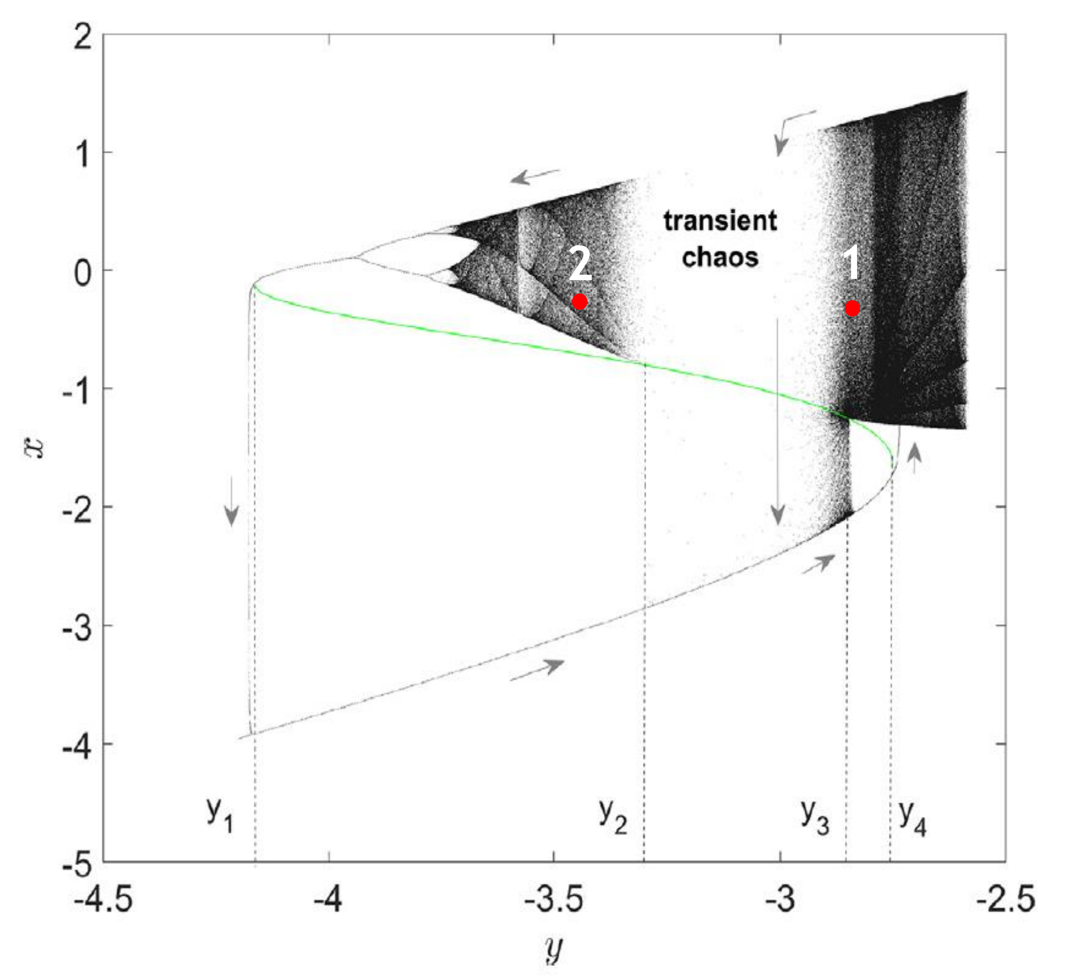
$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n$$
$$y_{n+1} = y_n - \sigma x_n - \beta,$$

$\alpha = 4.1$

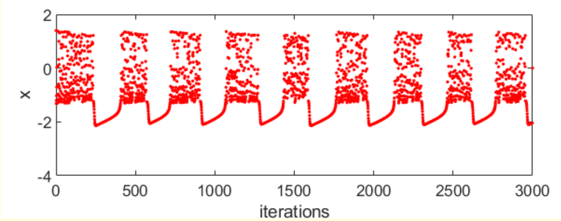
$\sigma = \beta = 0.001$

Fast variable

Slow variable



1



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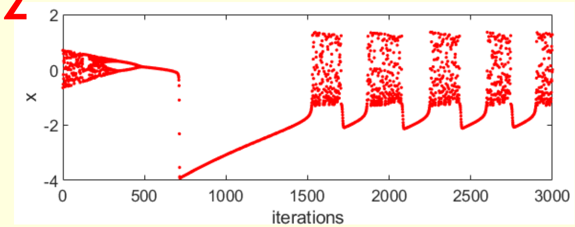
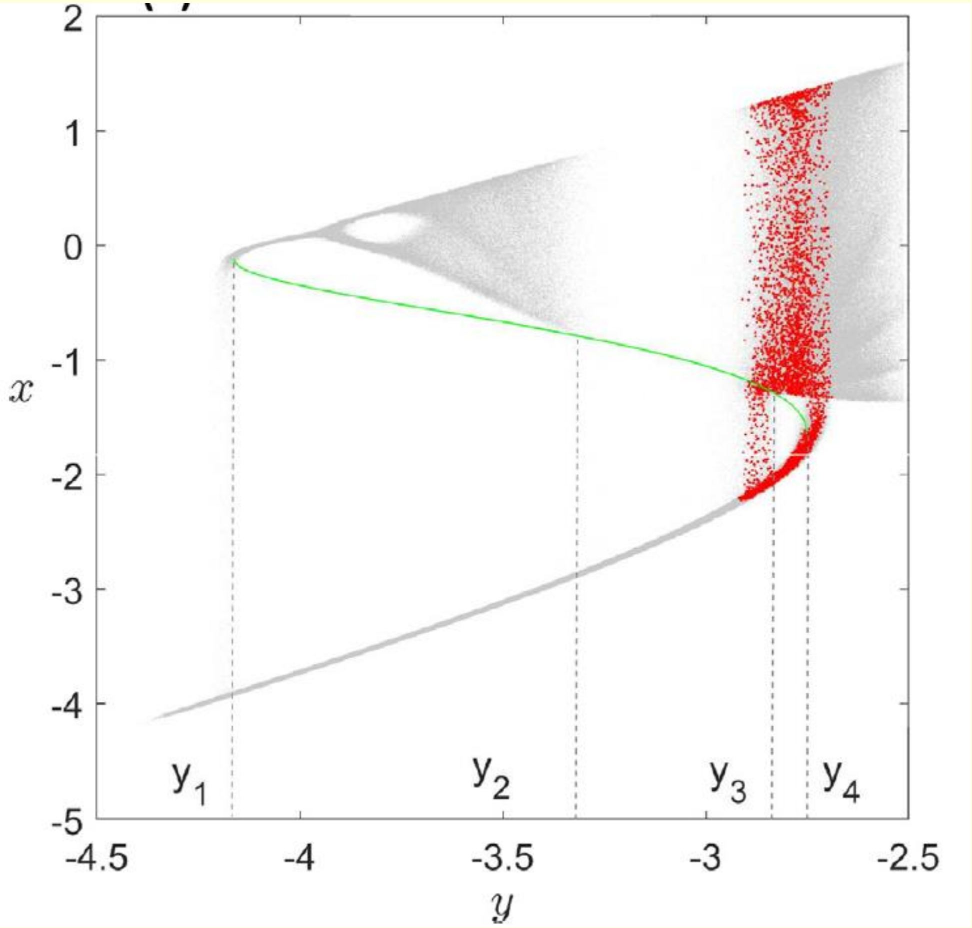


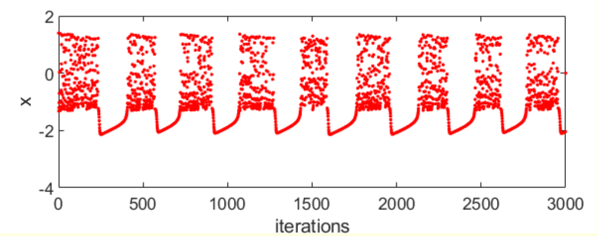
Adding noise



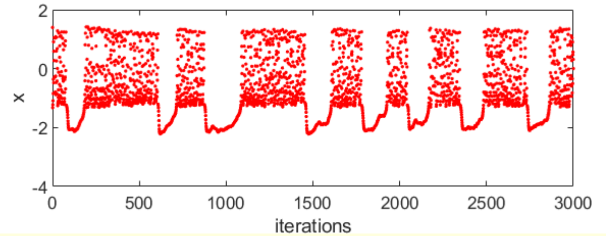
$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n + \xi_n^x$$
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y,$$

$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0$$

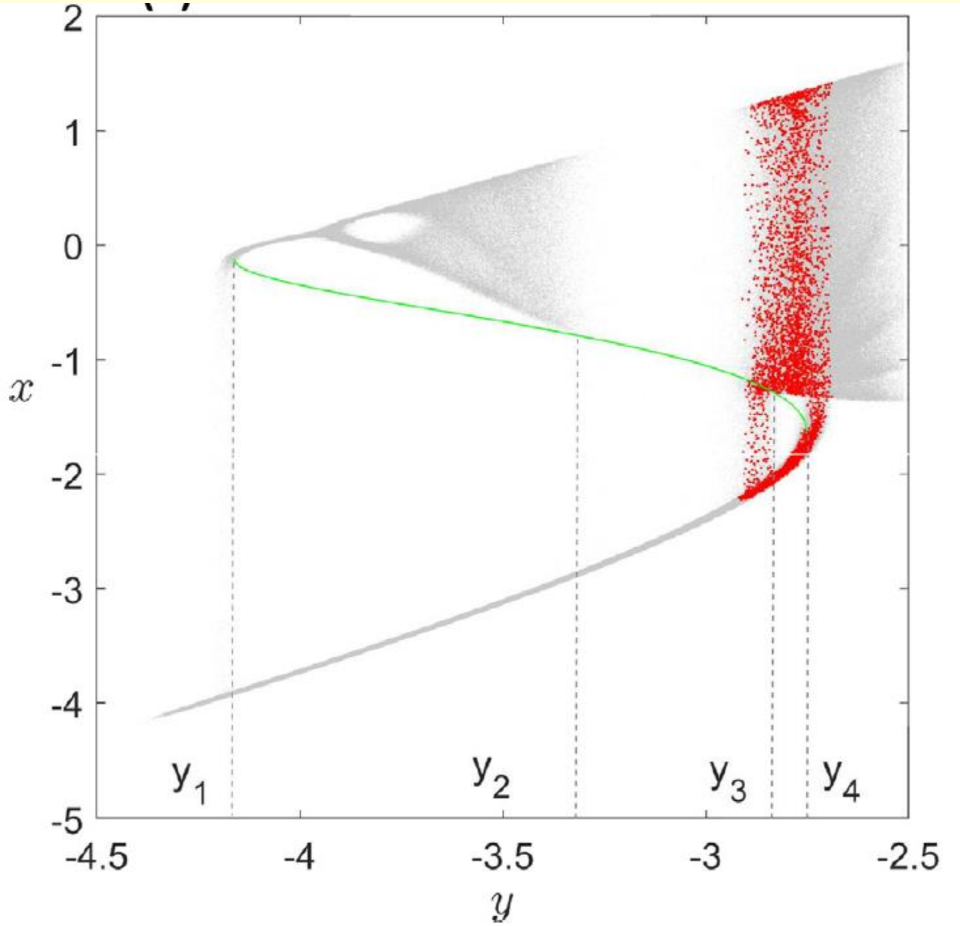
No noise



With bounded noise



Control goal: increase the bursting size



$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n + \xi_n^x + u_n^x$$

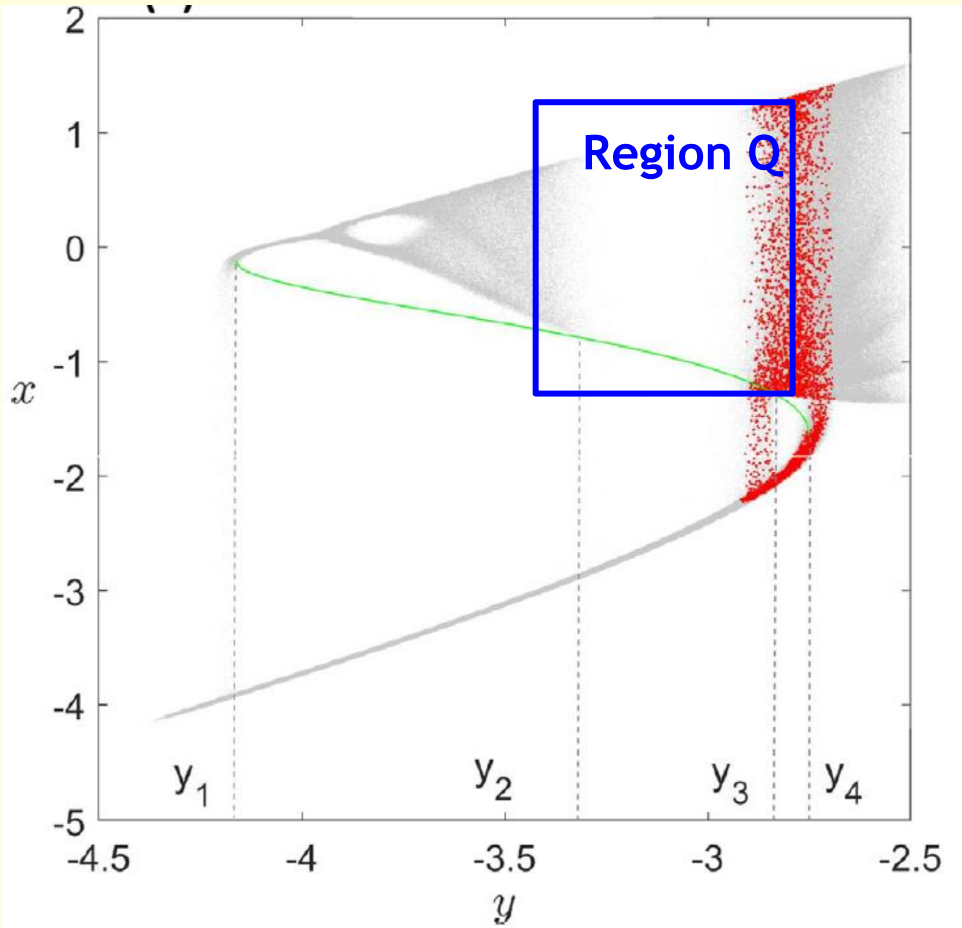
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

map   noise   control

$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0$$

$$\sqrt{(u_n^x)^2 + (u_n^y)^2} \leq u_0$$

Control goal: increase the bursting size



$$x_{n+1} = \frac{\alpha}{(1 + x_n^2)} + y_n + \xi_n^x + u_n^x$$

$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

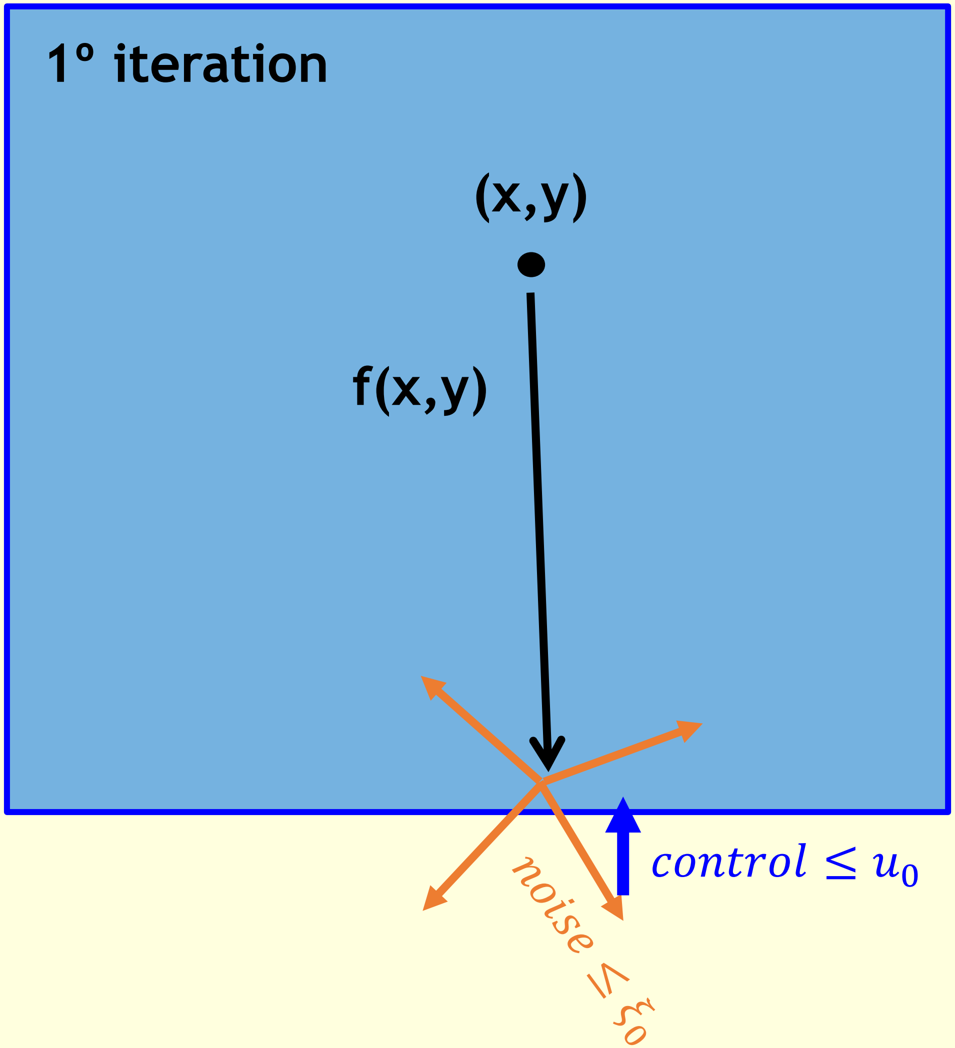
map noise control

$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0$$

$$\sqrt{(u_n^x)^2 + (u_n^y)^2} \leq u_0$$

Sculpting algorithm

Region Qo



$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n + \xi_n^x + u_n^x$$
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

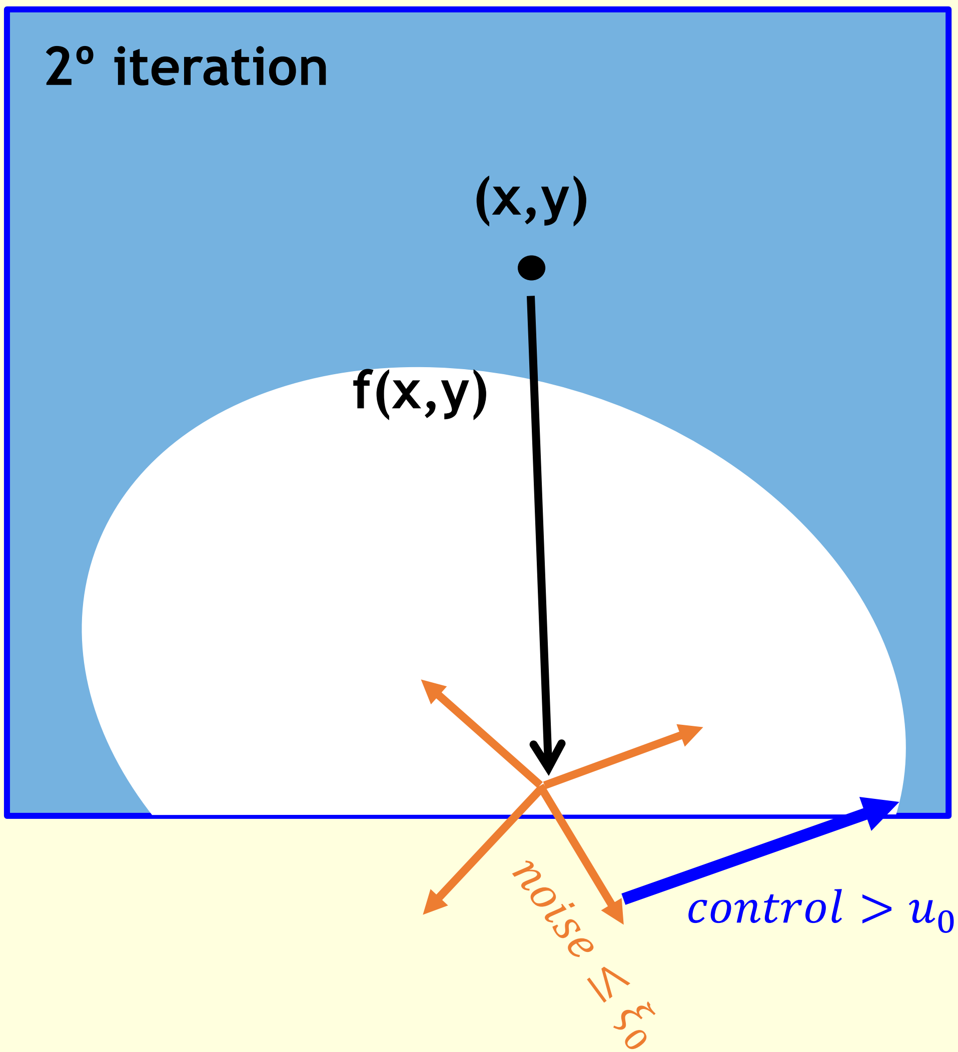
map — noise — control

$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0$$

$$\sqrt{(u_n^x)^2 + (u_n^y)^2} \leq u_0$$

Sculpting algorithm

Region Q1



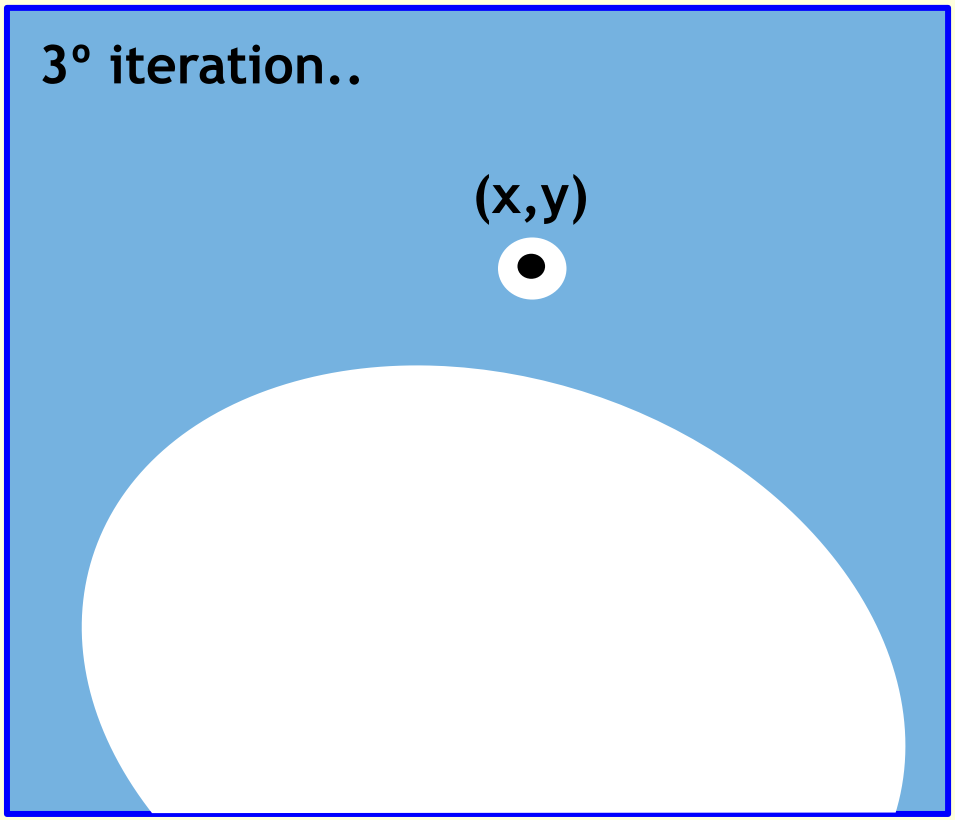
$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n + \xi_n^x + u_n^x$$
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

map noise control

$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0 \qquad \sqrt{(u_n^x)^2 + (u_n^y)^2} \leq u_0$$

Sculpting algorithm

Region Q2



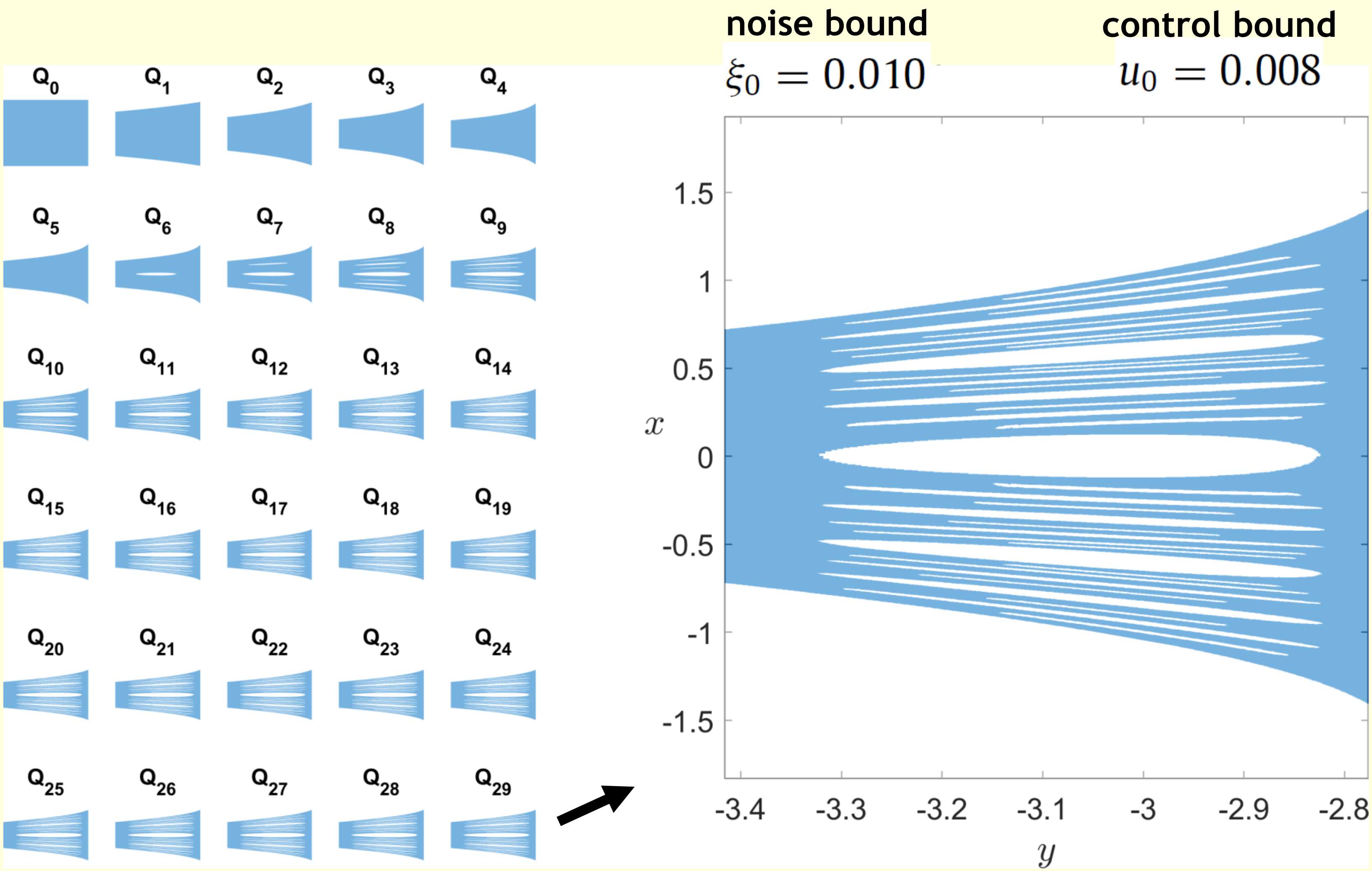
$$x_{n+1} = \frac{\alpha}{(1+x_n^2)} + y_n + \xi_n^x + u_n^x$$
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

map — noise — control

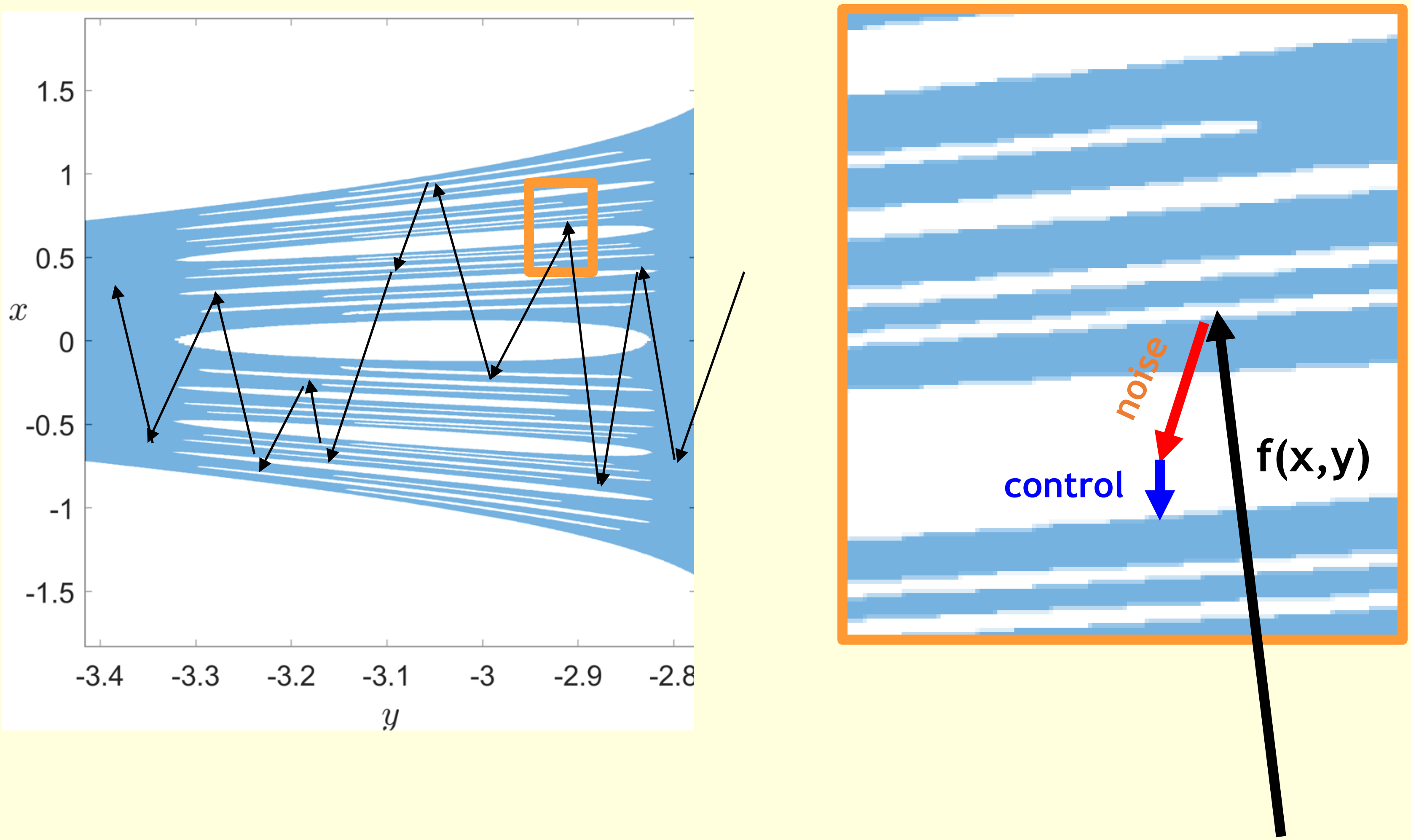
$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \le \xi_0$$

$$\sqrt{(u_n^x)^2 + (u_n^y)^2} \le u_0$$

Sculpting algorithm: final set

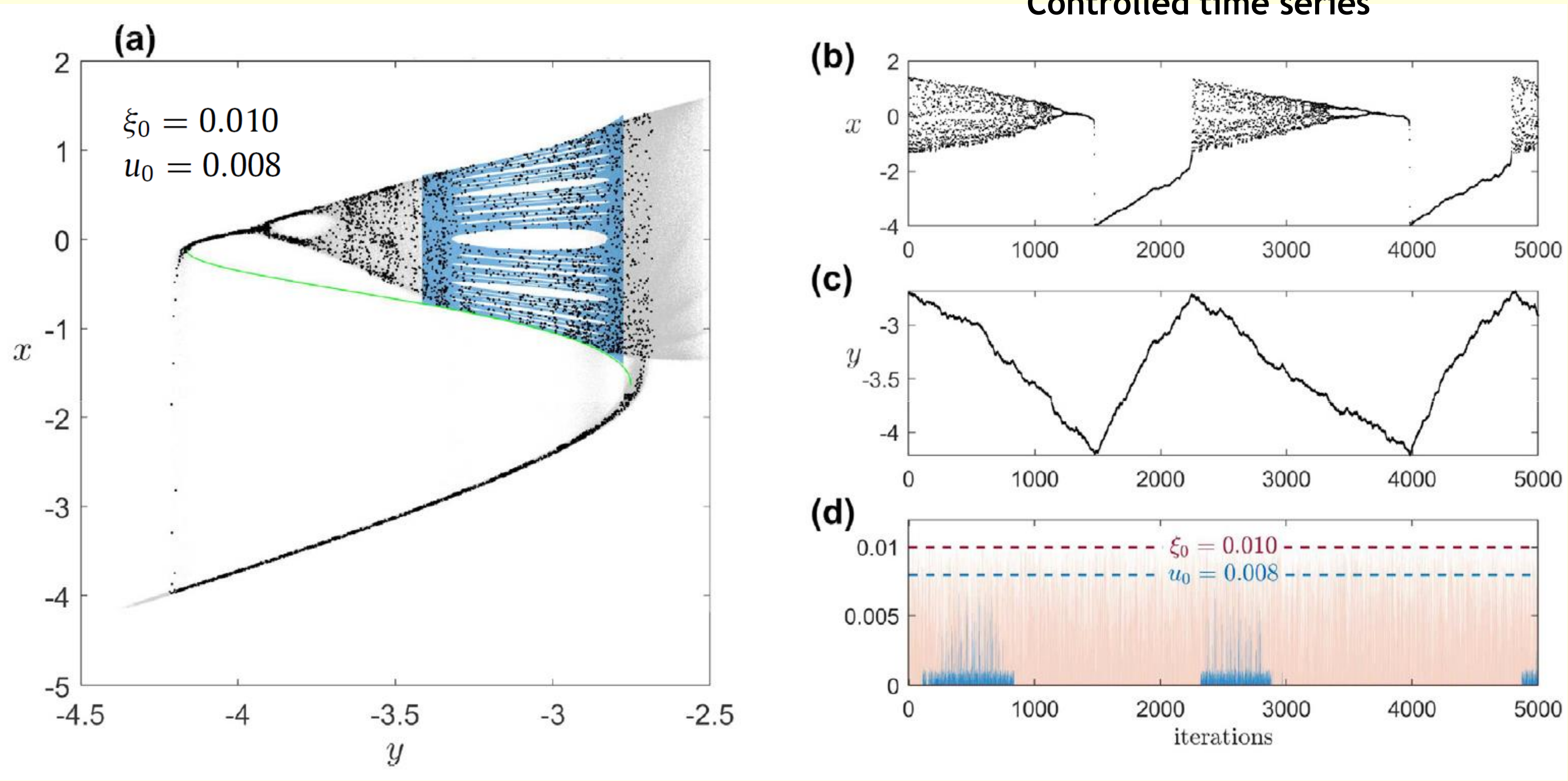


Control in the final set



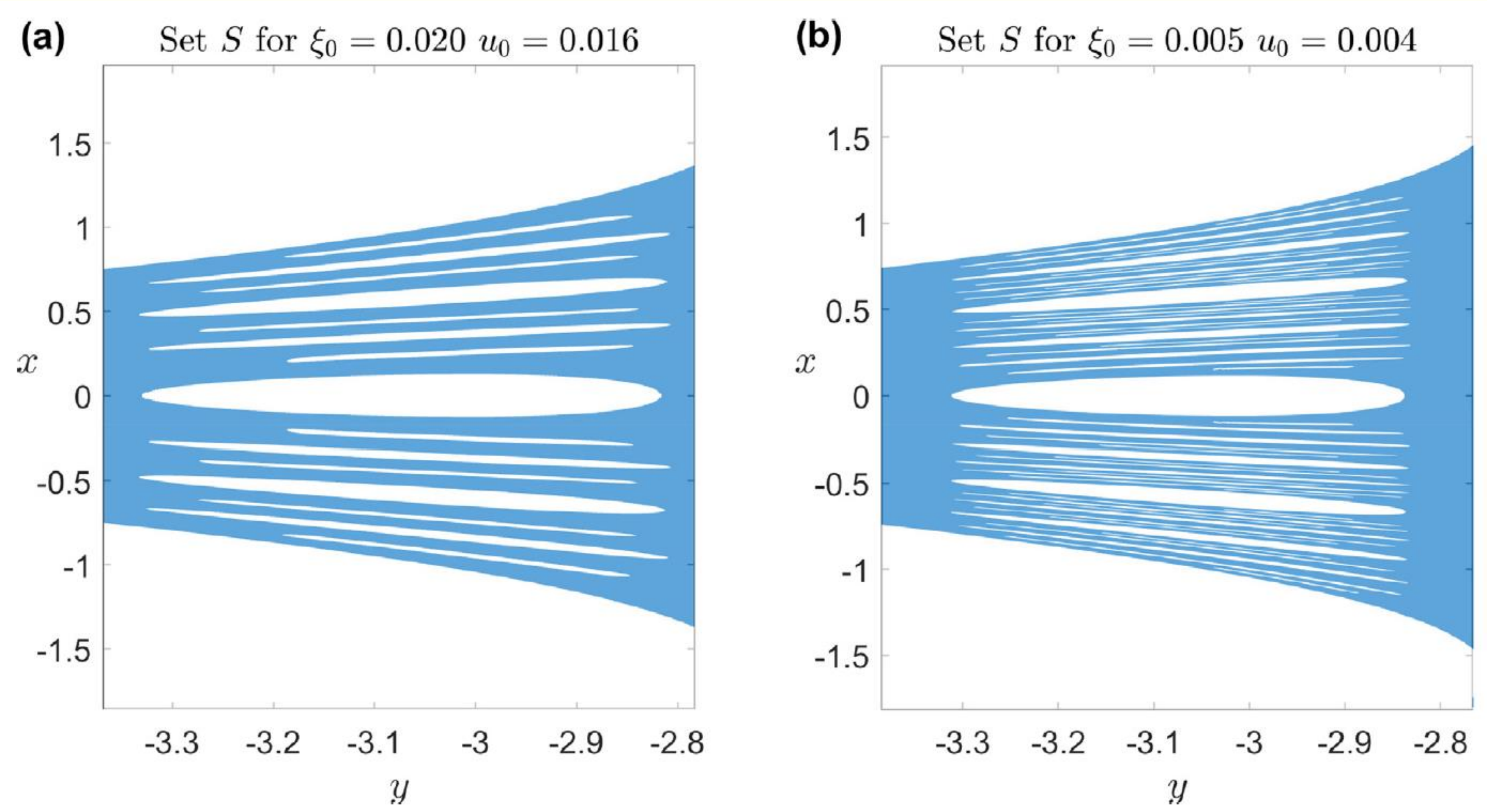
Control:long bursting

Controlled time series



12/16

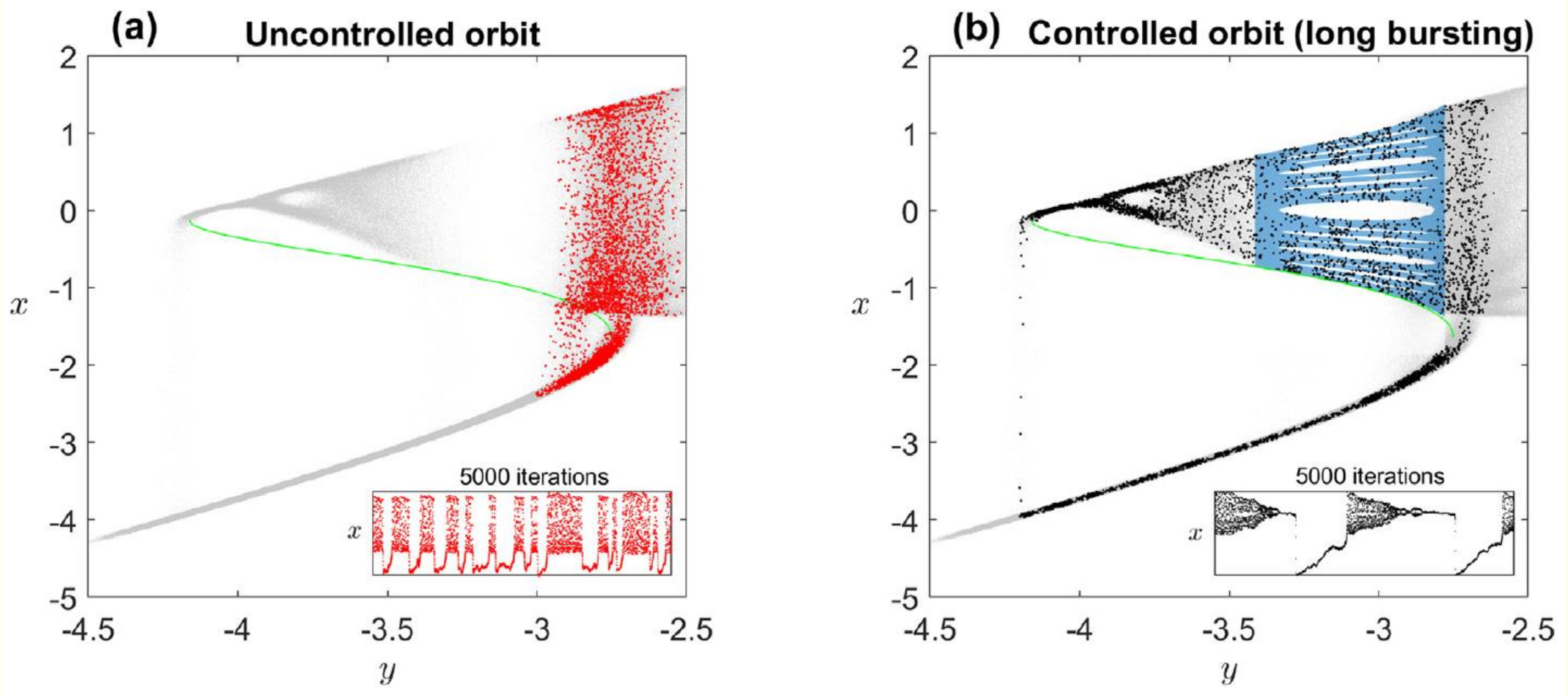
Different noise ➞ Different sets



13/16

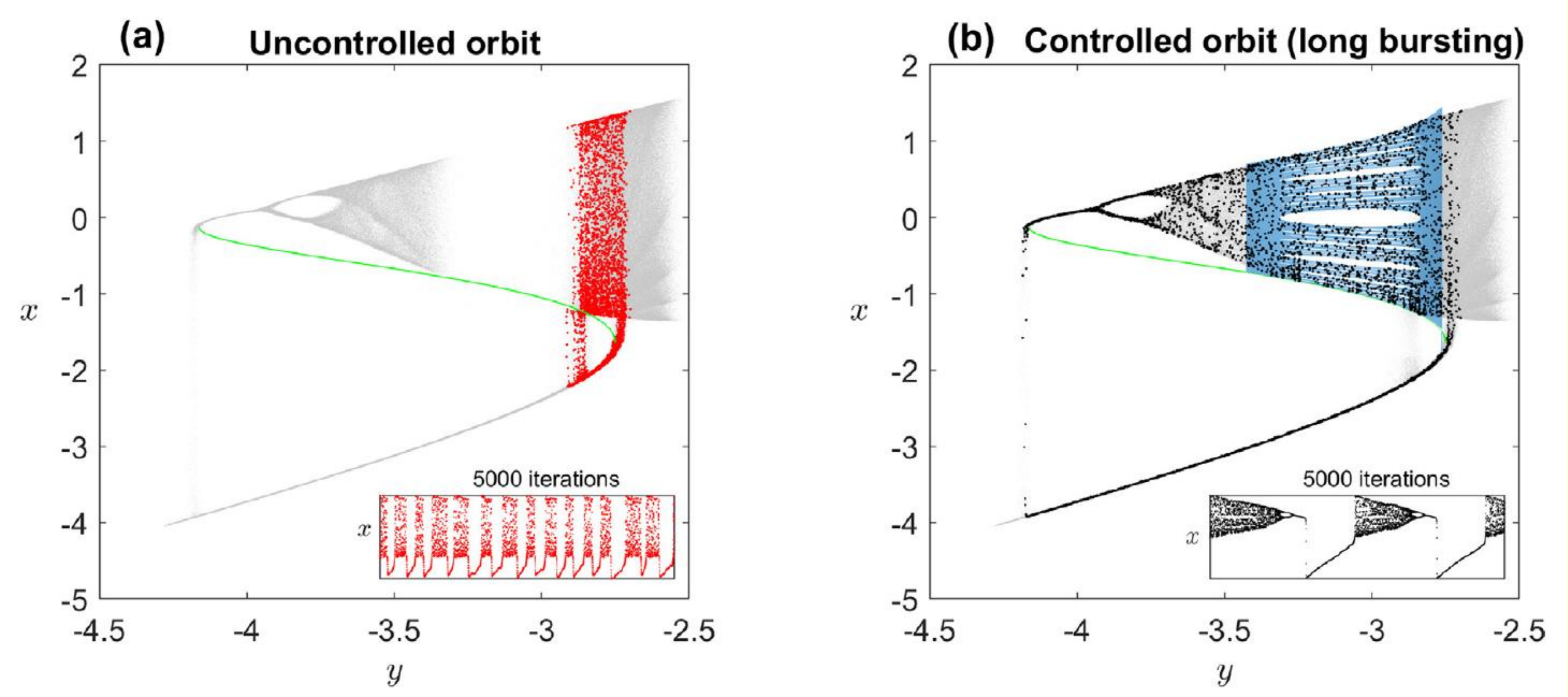
Different noise ➞ Different sets

Set S for $\xi_0 = 0.020$ $u_0 = 0.016$



Different noise ➔ Different sets

Set S for $\xi_0 = 0.005$ $u_0 = 0.004$



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Research paper

Controlling the bursting size in the two-dimensional Rulkov model

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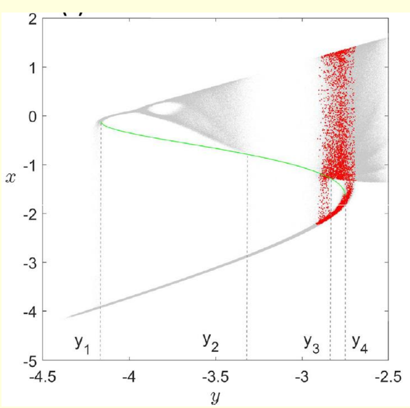
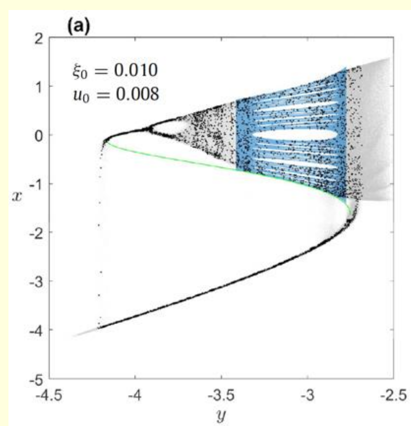
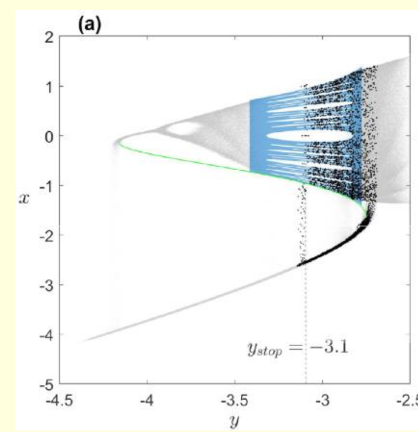
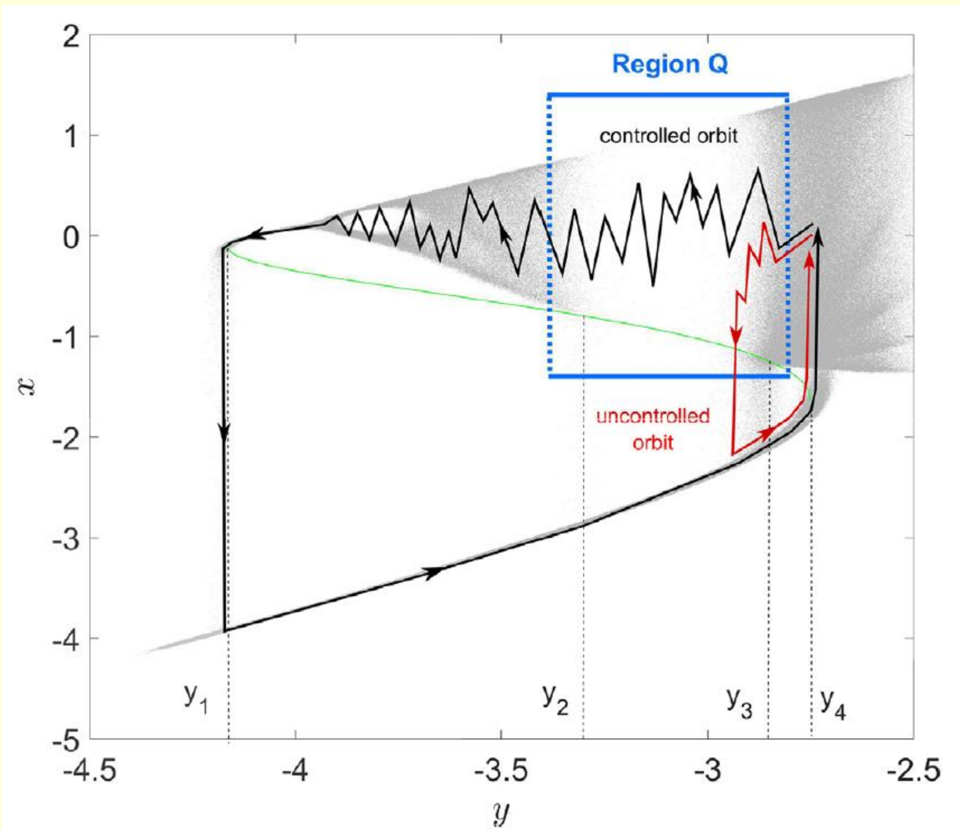






Control Goal



$$x_{n+1} = \frac{\alpha}{(1 + x_n^2)} + y_n + \xi_n^x + u_n^x$$
$$y_{n+1} = y_n - \sigma x_n - \beta + \xi_n^y + u_n^y,$$

map noise control

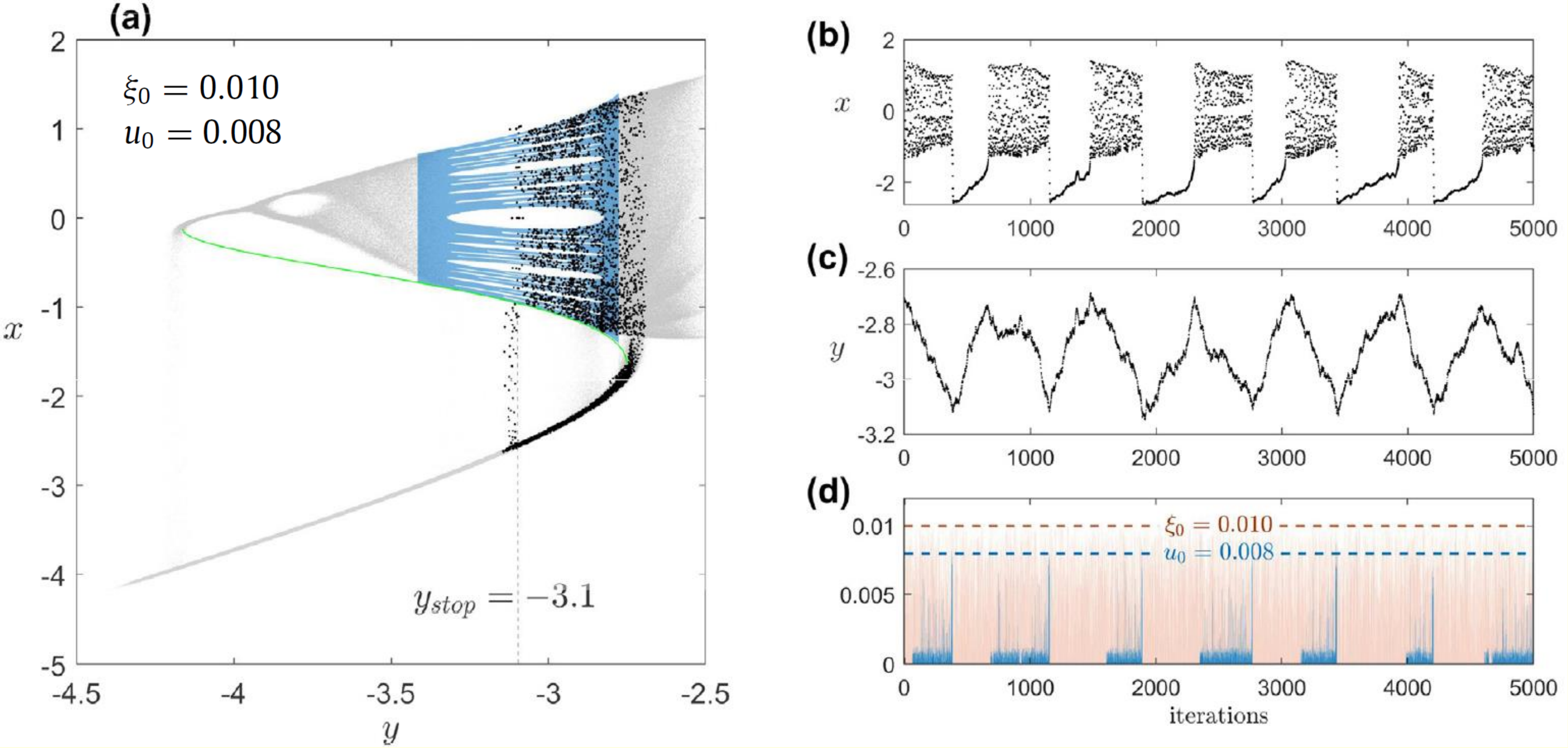
$$\sqrt{(\xi_n^x)^2 + (\xi_n^y)^2} \leq \xi_0$$

$$\sqrt{(u_n^x)^2 + (u_n^y)^2} \leq u_0$$

Control:stopped bursing

Two control options: -natural escape

-push the orbit out

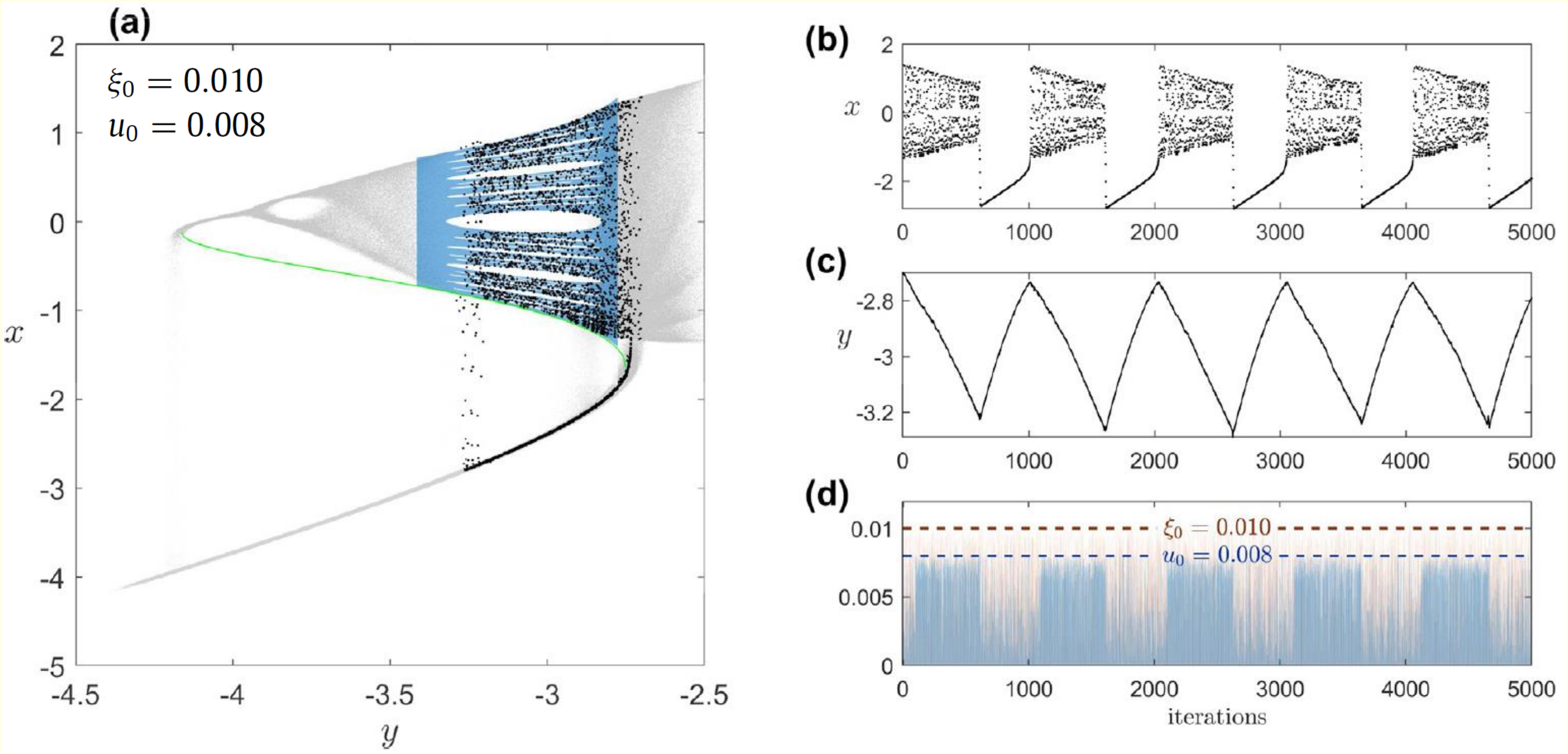


18/16

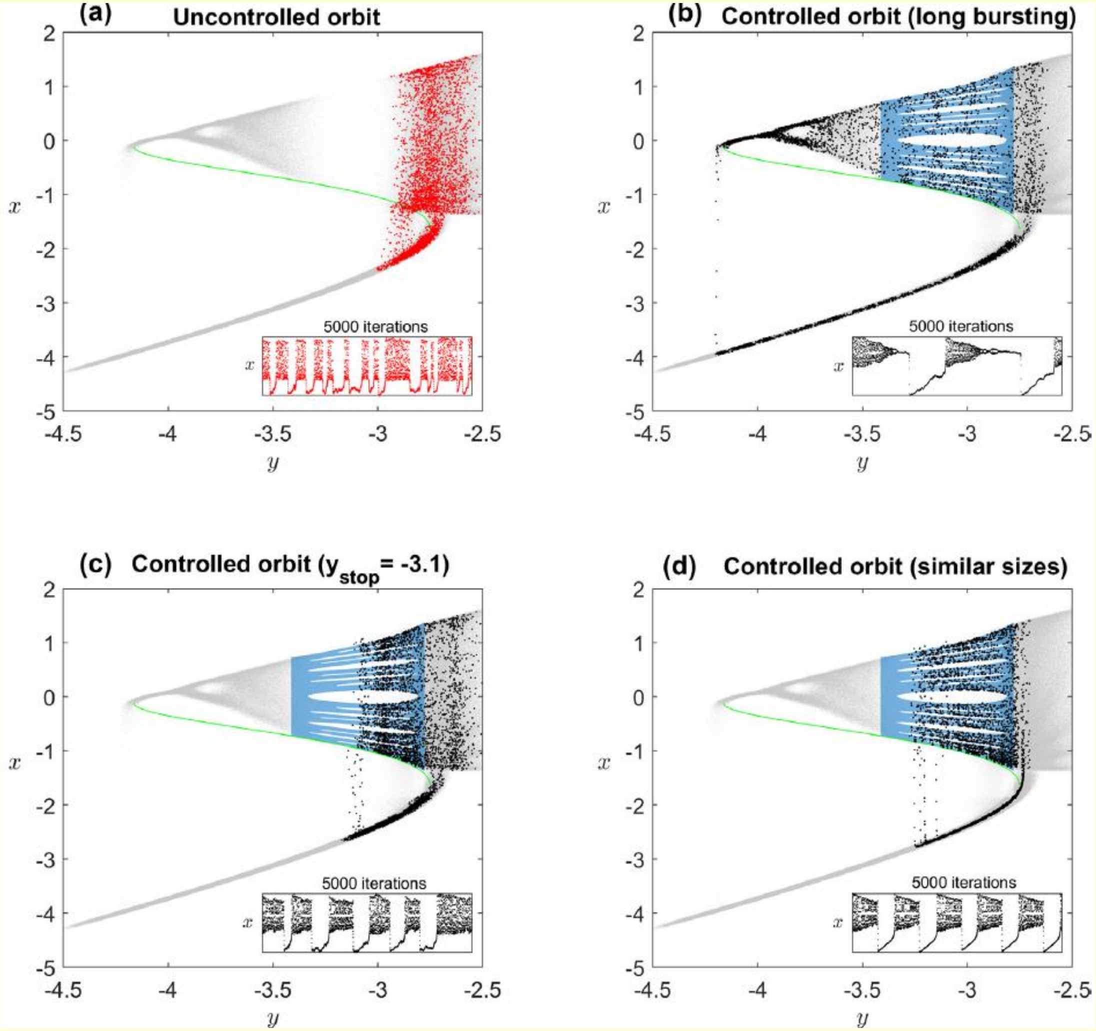
Control: similar bursting size

Additional control over 'y' to follows the deterministic 'y'

Each 5000 iterations the control ceases

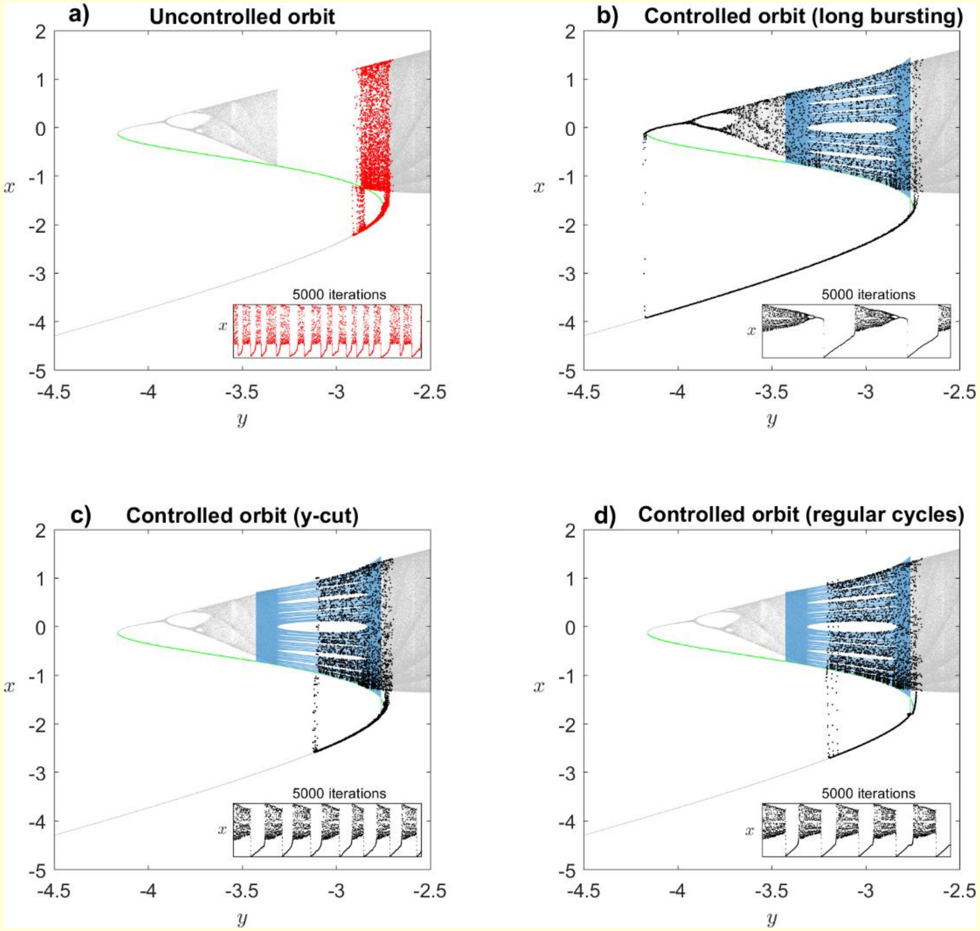


Transient chaos



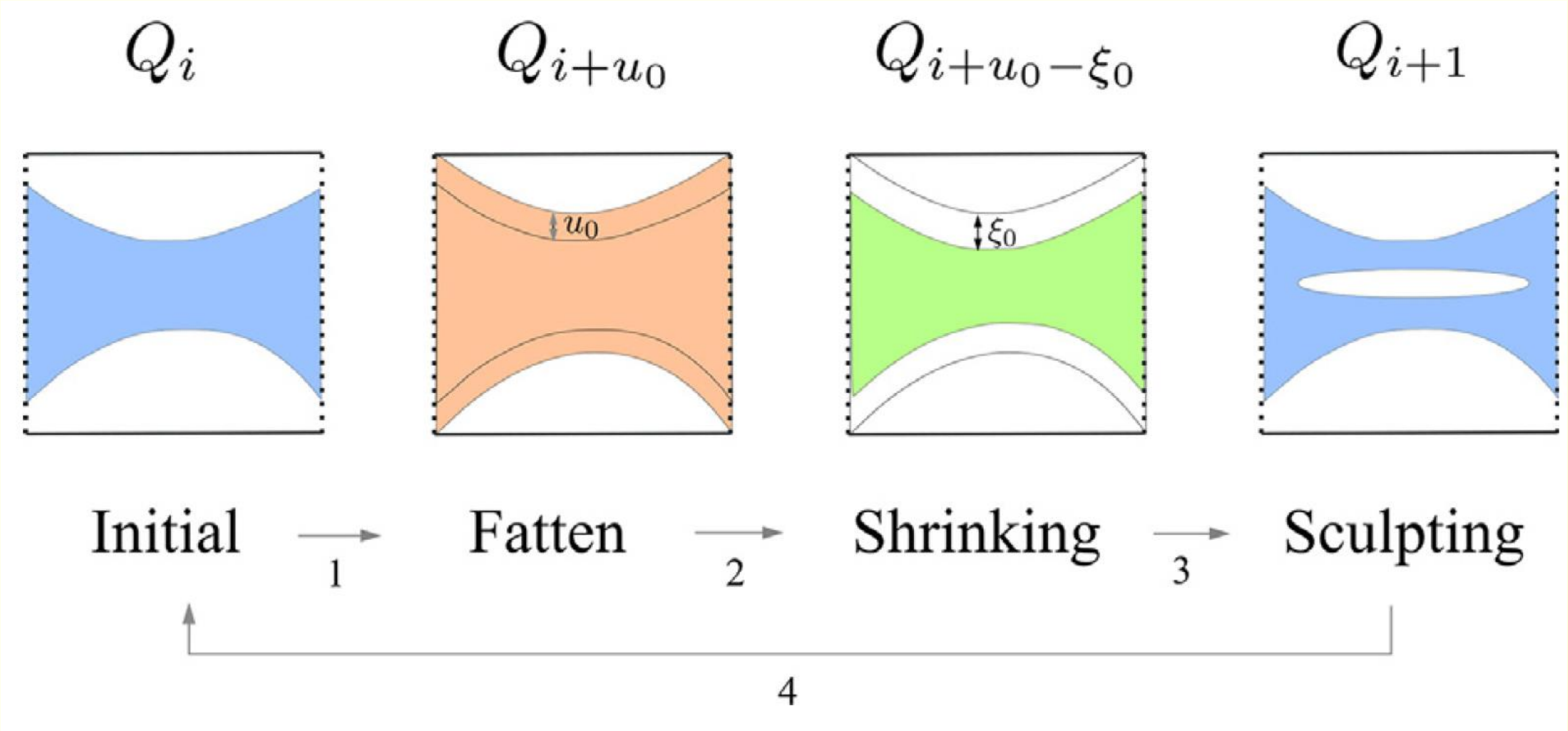
20/16

Transient chaos



21/16

Transient chaos

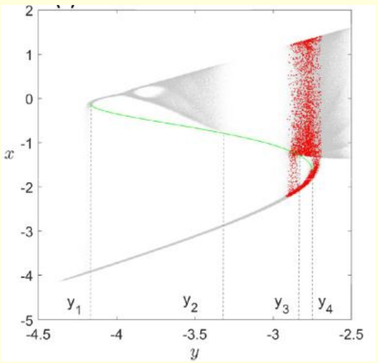


Step 1. Fatten the set Q_i by u_0 except the right and left boundaries, obtaining the set denoted by $(Q_i + u_0)$.
Step 2. Shrink the set $(Q_i + u_0)$ by ξ_0 except the right and left boundaries, obtaining the set denoted by $(Q_i + u_0 - \xi_0)$.
Step 3. Let Q_{i+1} be the points $q \in Q_i$, for which $f(q)$ fall inside the set denoted $(Q_i + u_0 - \xi_0)$, or the points $q \in Q_i$ for which $f(q)$ abandon Q through the right or left boundaries.
Step 4. Return to step 1, unless $Q_{i+1} = Q_i$. We call this final region, the set S.

Conclusions

- The control method presented is applied on maps that exibits transient chaotic dynamics and are affected by noise.
- The control is applied with the goal to sustain the orbit in certain región Q of the phase space.
- To apply we need to define Q, the bound of noise and the bound of control applied. Through an iterative algorithm the región Q is sculpted to obtain a subset S where the orbits are controlled.

uncontrolled



controlled