On the Product Selection and Plant Dimensioning Problem under uncertainty[*]

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Abstract

We present a two-stage full recourse model for strategic production planning under uncertainty, whose aim consists of determining product selection and plant dimensioning. The main uncertain parameters are the product price, demand and production cost. The benefit is given by the product net profit over the time horizon minus the investment depreciation and operation costs. The Value-at-Risk and the reaching probability are considered as risk measures in the objective function to be optimized as alternatives to the maximization of the expected benefit over the scenarios. The uncertainty is represented by a set of scenarios. The problem is formulated as a mixed 0–1 Deterministic Equivalent Model. The strategic decisions to be made in the first stage are represented by 0–1 variables. The tactical decisions to be made in the second stage are represented by continuous variables. An approach for problem solving based on a splitting variable mathematical representation via scenario is considered. The problem uses the Twin Node Family concept within the algorithmic framework known as Branch-and-Fix Coordination for satisfying the nonanticipativity constraints. Some computational experience is reported.

Keywords: Production planning, mean-risk, stochastic programming, mixed 0-1 programs, splitting variable, branch-and-fix coordination.

1   Introduction

Given a time horizon and a set of market sources, the *Product Selection and Plant Dimensioning Problem* (*PSPDP*) is concerned with determining what products to produce and to market, and with the plant selection and production capacity dimensioning at the beginning of the time horizon. The aim is to maximize the expected product net profit minus the investment depreciation and operation cost, subject to the constraints related to budget limitations for plant dimensioning, production requirements and product demand from market sources.

The deterministic version of the problem can be represented as a mixed 0–1 program comprising (a) the set of the 0–1 variables related constraints for the production system topology, and (b) a dynamic structure for the multi-period production and demand related constraints. Given today's state-of-the-art optimization tools no major difficulties should arise for problem solving in moderate sized instances, at least. (In any case, we should always keep in mind that 0–1 programs are NP-hard problems).

However, the product price, demand and production cost are uncertain parameters at the investment decision time for long-range time horizons, at least. So *PSPDP* is an interesting case of the application of stochastic programming.

Most stochastic approaches to production planning only consider tactical decisions, usually modelled by continuous variables, see [9, 10]. On the other hand, there are a few schemes that address the strategic production planning under uncertainty, modelled by using 0–1 and continuous variables, see [1, 3, 15], among others. However, see [4] for a survey on mixed 0–1 problem modelling for strategic and tactical planning, and job scheduling and operation sequencing under uncertainty. Stochastic 0–1 Programming has a broad application field and is flourishing (see particularly the books [21, 23, 24], among others). In any case, most of these approaches only consider mean (expected) objective functions. Very few approaches deal with mean-risk measures by considering semi-deviations [17] and excess probabilities [19], see also [8, 14, 20, 22]. These approaches are more amenable for large-scale problem solving than the classical mean-variance schemes, mainly in the presence of 0–1 variables.

In this paper we present a mixed 0–1 *Deterministic Equivalent Model* (*DEM*) for the two-stage stochastic *PSPDP* with full recourse, where parameter uncertainty is represented by a set of scenarios. The first stage variables are (strategic) 0–1 variables to determine product selection and plant dimension. The second stage variables are (tactical) continuous variables to determine the production, stock and market shipment of the products for each period throughout the time horizon under each scenario. The values of the strategic variables consider all scenarios without being subordinated to any of them. The *Value-at-Risk (VaR)* and the *reaching probability* are considered as risk measures in contrast to approaches where only mean functions appear in the objective function to be optimized.

Given the combinatorial nature of the problem and the large dimensions of the instances, (basically, due to the number of scenarios), it is not realistic to seek the optimal solution in a stochastic environment without exploiting the stochastic problem's structure, even when using state-of-the-art optimization engines.

We present a *Branch-and-Fix Coordination* (*BFC*) scheme to exploit the structure of the *splitting variable* representation of the mixed 0–1 *DEM* and, specifically, the structure of the *nonanticipativity* constraints for the 0–1 variables. The algorithm makes use of the *Twin Node Family* concept introduced in [2, 3]. It is especially designed for coordinating node branching selection and node pruning and 0–1 variable branching selection and fixing at each scenario-related *Branch-and-Fix* (*BF*) tree. The computational experience obtained so far shows that the proposed *BFC* scheme provides the optimal solution for cases where plain utilization of the optimization engine IBM OSL v2.0 does not and, in any case, the computational effort is smaller (very frequently, by one order of magnitude, at least).

The remainder of the paper is organized as follows. Section 2 states the *PSPDP* and introduces the mixed 0–1 *DEM* for the two-stage stochastic version of the problem. Section 3 presents the *BFC* approach to problem solving. Section 4 reports on computational experience with different priority strategies for the *BFC* approach, and makes a comparison with the plain optimization of the *DEM* and the traditional approach based on the average scenario deterministic model. Section 5 concludes. A detailed *BFC* procedure is given in the Appendix.

2 Description of the Problem

2.1 Mixed 0–1 Deterministic Equivalent Model (*DEM*)

Consider a problem that has the following elements. A *time horizon* is a set of (consecutive and integer) time periods for production planning. A *product* is any item whose selection and production volume is decided by management. The *stock* of a product is its available volume at the end of a given time period. Assume that the cycle time (i.e. lead time) of any unit product is smaller than the length of the periods in the time horizon.

A *plant* is a capacitated location where products are processed. Plants may have different production capacity levels. The term *plant investment* will be used for the amount of a given currency that is needed to expand a plant from, say, level $k-1$ to level k. Observe that expansion to level $k = 1$ means that a plant will be open. It is assumed that plants are designed to process specific products.

Some parameters are deterministic by nature or the optimal solution may not be very sensitive to their variability. However, product net profit and demand and production cost are uncertain parameters, mainly for long time horizons, as is usually the case for strategic planning. The information available for the uncertain parameters can be structured in a set of *scenarios*.

The aim of the strategic production planning problem that is considered in this paper is to determine the production topology, i.e., product selection and plant dimensioning. The objective is to maximise a composite function given by the expected benefit (in constant terms) and the probability of reaching a benefit target. Alternatively, the *VaR* can be maximized for a given probability. The benefit can be expressed as the product net profit minus the production costs and the plant investment depreciation cost over the time horizon for all scenarios.

Two *stages* are considered. The first stage is devoted to *strategic* decisions on product selection and plant dimensioning. The second stage is devoted to *tactical* decisions about the volume of products to be processed and stored in plants and the volume to be shipped from plants to market sources in each period over the time horizon for all scenarios, given the production topology decided at the first stage. Besides satisfying their related first stage constraints, strategic decisions take into account the expected net profit from products and expected production cost related to the tactical environment and investment depreciation cost. We have to consider that the problem presented above is an abstraction of reality since, among other reasons, (a) product selection and plant (re)dimensioning take place in reality throughout the time horizon and not only at the first stage, and (b) there are higher level reasons than tactical ones, budget considerations and depreciation costs alone to decide on strategic issues. However, the model dealt with in the paper illustrates how some essential aspects of real-life problems can be treated for problem solving.

See [3] for a generalization of the model considered below to a multilevel production environment and vendor selection for raw material.

The following is the notation for the elements of the strategic production planning model. *Sets:*

$\mathcal{I}$, set of plants.

$\mathcal{J}$, set of products.

$\mathcal{J}_i$, set of products that can be processed in plant i, for $i \in \mathcal{I}$, ($\mathcal{J}_i \subseteq \mathcal{J}$).

$\mathcal{K}_i$, set of production capacity levels for plant i, for $i \in \mathcal{I}$.

$\mathcal{M}_j$, set of market sources for product j, for $j \in \mathcal{J}$.

$\mathcal{T}$, set $\{1, \dots, |\mathcal{T}|\}$ of periods throughout the time horizon (i.e. second stage).

Technical and logistic parameters:

N^δ, maximum number of plants to be opened.

N^γ, maximum number of products to be processed.

P, available budget for plant capacity dimensioning. Note: It is assumed that this can only occur in time period $t = 0$.

$\underline{X}_{jt}, \overline{X}_{jt}$, conditional minimum and maximum volume of product j that can be processed in time period t, for $j \in \mathcal{J}, t \in \mathcal{T}$.

q_j^i, unit capacity usage of plant i by product j, for $j \in \mathcal{J}_i, i \in \mathcal{I}$.

$\underline{p}_i$, minimum capacity usage of plant i in any time period, if any, for $i \in \mathcal{I}$.

p_i^k, production capacity increment from level $k-1$ to level k in plant i, for $k \in \mathcal{K}_i, i \in \mathcal{I}$.

Deterministic cost coefficients:

e_i^k, budget required to expand production capacity from level $k-1$ to level k in plant i, for $k \in \mathcal{K}_i, i \in \mathcal{I}$. Note: It is assumed that $e_i^{k-1} > e_i^k$.

a_i^k, depreciation cost (over the time horizon) of investment e_i^k related to the $k-$th production capacity level in plant i, for $k \in \mathcal{K}_i, i \in \mathcal{I}$.

h_j^i, unit holding cost of product j in plant i in any time period, for $j \in \mathcal{J}_i, i \in \mathcal{I}$.

d_{jt}^{im}, unit transport cost of product j from plant i to market source m in time period t, for $m \in \mathcal{M}_j, j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}$.

Scenario-related and uncertain parameters:

w^ω, weight factor assigned to scenario ω, for $\omega \in \Omega$, such that $\sum_{\omega \in \Omega} w^\omega = 1$.

$D_{jt}^{m\omega}$, demand for product j from market source m at time period t under scenario ω, for $m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega$.

$b_{jt}^{im\omega}$, unit net profit from selling product j from plant i to market source m in time period t under scenario ω, for $m \in \mathcal{M}_j, j \in \mathcal{J}_i, i \in \mathcal{I}, t \in T, \omega \in \Omega$.

$c_{jt}^{i\omega}$, unit processing cost of product j in plant i in time period t under scenario ω, for $j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega$.

Several 0–1 equivalent formulations can be considered, see [3]. Our approach requires the following variables.

Strategic variables. These are 0–1 variables, such that

$$\gamma_j = \begin{cases} 1, & \text{if product } j \text{ is selected for processing} \\ 0, & \text{otherwise,} \end{cases} \qquad \forall j \in \mathcal{J}$$

$$\delta_i^k = \begin{cases} 1, & \text{if plant } i \text{ has production capacity level } k \\ 0, & \text{otherwise,} \end{cases} \qquad \forall k \in \mathcal{K}_i, i \in \mathcal{I}.$$

Tactical variables. These are continuous variables for each market source m, product j, plant i, time period t and scenario ω, for $m \in \mathcal{M}_j, j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega$.

$X_{jt}^{i\omega}$, volume of product j to be processed in plant i in time period t under scenario ω.

$S_{jt}^{i\omega}$, stock volume of product j in plant i in (at the end of) time period t under scenario ω.

$Y_{jt}^{im\omega}$, volume of product j to be shipped from plant i to market source m in time period t under scenario ω.

Let the following be a *compact* representation of the *DEM* for the two-stage stochastic problem with full recourse,

$$\max \quad \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \sum_{m \in \mathcal{M}_j} \sum_{t \in \mathcal{T}} w^\omega (b_{jt}^{im\omega} - d_{jt}^{im}) Y_{jt}^{im\omega}$$

$$- \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \sum_{t \in \mathcal{T}} w^\omega (h_j^i S_{jt}^{i\omega} + c_{jt}^{i\omega} X_{jt}^{i\omega})$$

$$- \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} a_i^k \delta_i^k \tag{2.1}$$

subject to

$$\sum_{i \in \mathcal{I}} \delta_i^1 \leq N^\delta \tag{2.2}$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} e_i^k \delta_i^k \leq P \tag{2.3}$$

$$\delta_i^k \leq \delta_i^{k-1} \qquad \forall k \in \mathcal{K}_i - \{1\}, i \in \mathcal{I} \tag{2.4}$$

$$\delta_i^1 \leq \sum_{j \in \mathcal{J}_i} \gamma_j \qquad \forall i \in \mathcal{I} \tag{2.5}$$

$$\gamma_j \leq \sum_{i \in I : j \in \mathcal{J}_i} \delta_i^1 \qquad \forall j \in \mathcal{J} \tag{2.6}$$

$$\sum_{j \in \mathcal{J}} \gamma_j \leq N^\gamma \tag{2.7}$$

$$\delta_i^k \in \{0,1\} \qquad \forall k \in \mathcal{K}_i, i \in \mathcal{I} \tag{2.8}$$

$$\gamma_j \in \{0,1\} \qquad \forall j \in \mathcal{J} \tag{2.9}$$

$$\underline{p}_i \delta_i^1 \leq \sum_{j \in \mathcal{J}_i} q_j^i X_{jt}^{i\omega} \leq \sum_{k \in \mathcal{K}_i} p_i^k \delta_i^k \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \tag{2.10}$$

$$\underline{X}_{jt} \gamma_j \leq \sum_{i \in I : j \in \mathcal{J}_i} X_{jt}^{i\omega} \leq \overline{X}_{jt} \gamma_j \qquad \forall j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega \tag{2.11}$$

$$S_{j,t-1}^{i\omega} + X_{jt}^{i\omega} = \sum_{m \in \mathcal{M}_j} Y_{jt}^{im\omega} + S_{jt}^{i\omega} \qquad \forall j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \tag{2.12}$$

$$\sum_{i \in \mathcal{I} : j \in \mathcal{J}_i} Y_{jt}^{im\omega} \leq D_{jt}^{m\omega} \gamma_j \qquad \forall m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega \tag{2.13}$$

$$S_{jt}^{i\omega} \geq 0 \qquad \forall j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \tag{2.14}$$

$$X_{jt}^{i\omega} \geq 0 \qquad \forall j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega \tag{2.15}$$

$$Y_{jt}^{im\omega} \geq 0 \qquad \forall m \in \mathcal{M}_j, j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega. \tag{2.16}$$

The objective function (2.1) consists of maximizing the expected benefit over the time horizon for all the scenarios. See below the system for maximizing a composite function given by the expected benefit and the reaching probability functional.

The first stage constraints are as follows: Constraint (2.2) ensures that the number of plants will not exceed the allowed maximum. Constraint (2.3) takes into account the plant investment budget. Constraints (2.4) ensure that the δ-variables are well defined. Constraints (2.5) force

the selection of one product, at least, of those that can be processed in a given plant, provided that it is open. Constraints (2.6) force at least one of those plants that can process a product to open, provided that it is selected. Notice that (2.5) and (2.6) are 0–1 redundant constraints; moreover, the system is *LP* tighter, so the efficiency of the proposed problem solving is increased. Constraint (2.7) bounds the number of products to be selected.

The second stage constraints for each period of the time horizon under each scenario are as follows: Constraints (2.10) force a conditional lower bound on plant usage and a maximum production limit. Constraints (2.11) force a conditional lower bound on the production volume of the products and a limitation on their maximum production. Constraints (2.12) are the stock balance conditions for the products. Constraints (2.13) ensure that the product shipped to market sources will not exceed the related demand; notice the *LP* tightening 0–1 redundancy of replacing D by $D\gamma$.

The *compact* representation (2.1)–(2.16) can be transformed into a *splitting variable* representation, such that the $\delta-$ and $\gamma-$variables are replaced by their siblings, say, the $\delta^{\omega}-$ and $\gamma^{\omega}-$variables for each scenario $\omega \in \Omega$. The *nonanticipativity* constraints (2.17) and (2.18) are appended to the model, for $\omega \in \Omega - \{1\}$.

$$\delta_i^{k\omega} - \delta_i^{k1} = 0 \qquad \forall k \in \mathcal{K}_i, i \in \mathcal{I} \qquad (2.17)$$

$$\gamma_j^{\omega} - \gamma_j^{1} = 0 \qquad \forall j \in \mathcal{J}. \qquad (2.18)$$

By relaxing (or, as the case may be, dualizing) the constraints (2.17) and (2.18), $|\Omega|$ independent scenario-related mixed 0–1 models result. See below.

2.2 Mean-risk and VaR objective functions

Let the following be a representation of the model (2.1)–(2.16),

$$
\begin{aligned}
Z_{IP} = &\max a\xi + \sum_{\omega \in \Omega} w^{\omega} c^{\omega} x^{\omega} \\
\text{s.t. } & A^1 \xi && = b^0 \\
& A^2 \xi + \quad Bx^{\omega} && = b^{\omega} \quad \forall \omega \in \Omega \\
& \xi \in \{0,1\}^n, x^{\omega} \geq 0 && \forall \omega \in \Omega,
\end{aligned}
\qquad (2.19)
$$

where a and and c^{ω} are the vectors of the objective function coefficients for scenario $\omega \in \Omega$; ξ is the $n-$vector of the first stage (0–1) $\delta-$ and $\gamma-$ variables; x^{ω} is the vector of the second stage (continuous) $X- S-$ and $Y-$variables for scenario $\omega \in \Omega$; A^1 and A^2 are the first stage and second stage constraint matrices related to the $\xi-$variables, respectively; B is the second stage constraint matrix related to the $x^{\omega}-$variables; and b^0 and b^{ω} are the right-hand-side (*rhs*) vectors for the first stage and second stage constraints for scenario $\omega \in \Omega$, respectively; all parameters with conformable dimensions.

The *splitting variable* representation via scenario of model (2.19) is as follows,

$$
\begin{aligned}
Z_{IP} = &\max \sum_{\omega \in \Omega} w^{\omega}(a\xi^{\omega} + c^{\omega} x^{\omega}) \\
\text{s.t. } & A^1 \xi^{\omega} && = b^0 && \forall \omega \in \Omega \\
& A^2 \xi^{\omega} + Bx^{\omega} && = b^{\omega} && \forall \omega \in \Omega \\
& \xi^{\omega} - \xi^1 && = 0 && \forall \omega \in \Omega - \{1\} \\
& \xi^{\omega} \in \{0,1\}^n, x^{\omega} \geq 0 && && \forall \omega \in \Omega,
\end{aligned}
\qquad (2.20)
$$

where the so-called *nonanticipativity* constraints are

$$\xi^{\omega} - \xi^1 = 0, \quad \forall \omega \in \Omega - \{1\}. \qquad (2.21)$$

The expected benefit function in (2.20) can be expressed as

$$Q_E = \sum_{\omega \in \Omega} w^\omega (a\xi^\omega + c^\omega x^\omega). \tag{2.22}$$

The probability of reaching a benefit that is not smaller than the prescribed target, say, ϕ over the scenarios (the so-called *reaching probability*) can be expressed as

$$Q_P = P\big(\omega \in \Omega : a\xi^\omega + c^\omega x^\omega \geq \phi\big). \tag{2.23}$$

So, alternatively to $\max Q_E$, the mean-risk function to optimize is as follows:

$$\max Q_E + \rho Q_P, \tag{2.24}$$

where ρ is a nonnegative weighting parameter. A more amenable expression of (2.24) (see [19, 20]) for computational purposes, at least, may be

$$
\begin{aligned}
\max \ &Q_E + \sum_{\omega \in \Omega} w^\omega \zeta^\omega \\
\text{s.t. } &a\xi^\omega + c^\omega x^\omega \geq \phi \zeta^\omega \quad \forall \omega \in \Omega \\
&\zeta^\omega \in \{0,1\} \quad\quad\quad \forall \omega \in \Omega,
\end{aligned} \tag{2.25}
$$

where ζ^ω is a 0–1 variable, such that

$$
\zeta^\omega = \begin{cases} 1, & \text{if the benefit under scenario } \omega \text{ is not smaller than } \textit{target } \phi \\ 0, & \text{otherwise,} \end{cases} \quad \forall \omega \in \Omega.
$$

Alternatively to $\max Q_E$ and $\max Q_E + \rho Q_P$, the *Value-at-Risk (VaR)* function to be optimized for a given α–risk, where $0 \leq \alpha < 1$, can be expressed as

$$
\begin{aligned}
\max \ &VaR \\
\text{s.t. } &a\xi^\omega + c^\omega x^\omega + M(1 - \zeta^\omega) \geq VaR \quad \forall \omega \in \Omega \\
&\sum_{w \in \Omega} w^\omega \zeta^\omega \geq 1 - \alpha \\
&\zeta^\omega \in \{0,1\} \quad\quad\quad\quad\quad\quad\quad\quad \forall \omega \in \Omega,
\end{aligned} \tag{2.26}
$$

where M is a parameter with the smallest value which does not eliminate any feasible configuration of the production topology.

Notice that the expressions for the *reaching probability* and the *VaR* can be considered as types of *chance* constraints (see [7]).

3 Branch-and-Fix Coordination algorithmic framework

We can see that the relaxation of constraints (2.21) in model (2.20) results in $|\Omega|$ independent models, such that each scenario-related model can be expressed as follows, for $\omega \in \Omega$,

$$
\begin{aligned}
Z_{IP}^\omega = \max \ &a\xi^\omega + c^\omega x^\omega \\
\text{s.t. } &A^1 \xi^\omega \quad\quad\quad\ = b^0 \\
&A^2 \xi^\omega + Bx^\omega \ = b^\omega \\
&\xi^\omega \in \{0,1\}^n, x^\omega \geq 0.
\end{aligned} \tag{3.1}
$$

The *BFC* approach is specially designed to coordinate the selection of the branching node and the branching variable for each scenario-related *BF* tree, such that the relaxed constraints (2.21) are satisfied when setting the appropriate variables to either one or zero. It also coordinates and reinforces the scenario-related *BF* node pruning, the variable fixing and the objective function bounding of the subproblems (3.1) attached to the nodes. See similar decomposition approaches in [6, 11, 12, 13, 16, 18, 19, 20], among others. However, those approaches focus more on using a Lagrangian relaxation of the constraints (2.21) to obtain good upper bounds, and less on branching and variable fixing. In any case, Lagrangian relaxation schemes can be added on top.

For the specialization of the *BFC* approach to solving problem (2.20), let $\mathcal{R}^\omega$ denote the *BF* tree associated with scenario ω, and $\mathcal{G}^\omega$ the set of active nodes in $\mathcal{R}^\omega$, $\omega \in \Omega$. Any two active nodes, say $g \in \mathcal{G}^\omega$ and $g' \in \mathcal{G}^{\omega'}$ are said to be *twin* nodes if either the paths from the *root* nodes to each of them in their own *BF* trees $\mathcal{R}^\omega$ and $\mathcal{R}^{\omega'}$, respectively, have branched on or been set to the same 0–1 values for the same variables ξ_i^ω and $\xi_i^{\omega'}$, for $\omega, \omega' \in \Omega$, $i \in \mathcal{I}$, or they are the same *root* nodes. A *Twin Node Family* (*TNF*), say, $\mathcal{H}_f$ is a set of nodes such that any one is a *twin* node to all the other members of the family, for $f \in \mathcal{F}$, where $\mathcal{F}$ is the set of *TNFs*. Note that $g, g' \in \mathcal{H}_f$ for any family $f \in \mathcal{F}$ implies that $\omega \neq \omega'$ for $g \in \mathcal{G}^\omega$ and $g' \in \mathcal{G}^{\omega'}$, $\omega, \omega' \in \Omega$. A *TNF* is labelled an integer one if all ξ–variables have 0–1 values and the constraints (2.21) are satisfied.

The replacement of the expected benefit function (2.22) by the mean-risk system (2.25) in (2.20) does not change the structure of the model, so the relaxation of the constraints (2.21) still allows it. On the other hand, the maximization of the *VaR* (2.26) destroys the structure of (2.20), since the constraint $\sum_{w \in \Omega} w^\omega \zeta^\omega \geq 1 - \alpha$ is not separable. However, notice that the Lagrangian relaxation that results from its dualization allows the separability of the model and, thus, the utilization of the *BFC* approach. We conjecture that its solution may still provide a good *VaR*.

BFC: Branch-and-Fix Coordination algorithm

Let us present the *BFC* algorithmic framework for solving model (2.20).

Step 1: Solve the *LP* relaxations of the $|\Omega|$ scenario-related models (3.1). Each model is attached to the *root* node in the trees $\mathcal{R}^\omega$, $\forall \omega \in \Omega$. If the models have 0–1 values for all the ξ–variables and the constraints (2.21) are satisfied then stop, the optimal solution to the original stochastic mixed 0–1 program (2.20) has been obtained. However, notice that those constraints have been relaxed in the *LP* models.

Step 2: The following parameters are saved in a centralized device called a *Master Program* (*MP*): the values of the variables and the solution value (i.e., the optimal objective function value) of the *LP* models attached to the (active) nodes in $\mathcal{G}^\omega$, $\forall \omega \in \Omega$, as well as the appropriate information for branching on the ξ–variables in the *TNFs* $\mathcal{H}_f$, $\forall f \in \mathcal{F}$. A decision is made in *MP* for: (a) selection of the *TNF* to branch, (b) selection of the branching variable and (c) variable setting across the *BF* trees. The decision is made available for the execution of each scenario-related *BF* phase.

Step 3: Optimization of the *LP* models attached to the newly created nodes from the members of the selected *TNF* by branching on the chosen ξ–variable.

Step 4: If the optimal solutions obtained in Step 3 have 0–1 values for all the ξ–variables and the constraints (2.21) are satisfied (i.e., the *TNF* is an integer one), a new solution has been found for the original program (2.20). The related *incumbent* solution can be updated and, additionally, the sets $\mathcal{G}^\omega$ at the trees $\mathcal{R}^\omega$, $\forall \omega \in \Omega$, are also updated. In any case, the *TNF* is pruned. The optimality of the *incumbent* solution has been proved if the sets of active nodes are empty. Otherwise, go to Step 2.

The Appendix gives a detailed description of a *BFC* procedure.

The proposed approach benefits from the heuristic scheme known as *HAFS*, to obtain an initial feasible solution whose objective function value, say $\underline{Z}$, will be used as a lower bound in obtaining the optimal solution by the *BFC* approach.

HAFS: Heuristic Algorithm for obtaining a Feasible Solution

Step 1: Obtain a solution for the δ– and γ–variables (i.e., a topology of the system, defining the product selection and plant dimensioning) that is guaranteed to be feasible for the constraint system (2.2)-(2.16) and, hopefully, can provide a good value for the objective function (2.1). For this purpose, the scheme exploits the structure of the problem and, thus, the fact that the uncertainty only relies on the demand parameters in the *rhs* and the coefficients on the objective function. So, the deterministic version of model (2.19) to solve can be expressed as:

$$
\begin{aligned}
Z = \max \, & a\xi + \overline{c}x \\
\text{s.t.} \ \ A^1\xi \quad & = b^0 \\
A^2\xi + Bx \ & = \overline{b} \\
\xi \in \{0,1\}^n, & \ x \geq 0,
\end{aligned}
\tag{3.2}
$$

where the components of the vector $\overline{c}$ are given by the subvector of the h–parameters and the average subvectors of the $(b-d)$– and c–parameters over the scenarios in the objective function (2.1); and the $\overline{b}$ is the *rhs* vector for the second-stage constraints, where the demand-related coefficients are given by the smallest demand, say $\underline{D}_{jt}^m$ of product j at time period t from market source m over the scenarios, $\forall m \in \mathcal{M}_j, j \in \mathcal{J}, t \in \mathcal{T}$, such that

$$
\underline{D}_{jt}^m = min_{\omega \in \Omega}\{D_{jt}^{m\omega}\}.
\tag{3.3}
$$

Step 2: Let the vector $\underline{\xi}$ denote the solution of model (3.2). A lower bound $\underline{Z}$ of the optimal solution of model (2.1)–(2.16) can be computed as

$$
\underline{Z} = \sum_{\omega \in \Omega} w^\omega Z^\omega,
\tag{3.4}
$$

where

$$
\begin{aligned}
Z^\omega \ = \ & a\underline{\xi} + \max c^\omega x^\omega \\
\text{s.t.} \quad & Bx^\omega = b^\omega - A^2\underline{\xi} \\
& x^\omega \geq 0.
\end{aligned}
\tag{3.5}
$$

4 Computational experience

We report the computational experience that we obtained while optimizing the *DEM* (2.1)–(2.16) by using the *BFC* approach presented in the previous section, for a randomly generated set of instances (see Table 1). Note: All experiments were carried out considering D instead of $D\gamma$ in constraints (2.13).

Our algorithmic approach was implemented in a FORTRAN experimental code. It uses the optimization engine IBM OSL v2.0 to solve the *LP* subproblems. The computational experiments were conducted on a WS Sun Park under the Solaris 2.5 operating system and using the FORTRAN 90 compiler.

Table 2 gives the dimensions of the scenario-related deterministic model (3.1) as well as the dimensions of the *DEM* (2.19) for the two-stage stochastic version, compact representation. The headings are as follows: $|\Omega|$, number of scenarios; m, number of constraints; nc, number of continuous variables; $n01$, number of 0-1 variables; and *dens*, constraint matrix density.

Table 1. Test bed dimensions

Case	$|\mathcal{J}|$	$|\mathcal{I}|$	$|\mathcal{K}|$	$|\mathcal{T}|$	$|\mathcal{M}|$	N^δ	N^γ
P1	7	5	3	5	3	4	6
P2	7	10	3	5	3	6	6
P3	9	8	3	5	4	6	6
P4	6	6	4	5	3	4	5
P5	5	5	4	5	3	4	4
P6	7	6	3	5	4	4	6
P7	7	5	3	5	3	3	5
P8	9	6	5	8	4	4	6
P9	4	5	3	5	3	3	3
P10	7	5	3	5	3	3	5
P11	4	5	3	5	3	4	4
P12	4	5	3	5	3	4	4
P13	4	5	3	5	3	4	4
P14	30	10	5	12	10	5	20

Table 2. Model dimensions

Case	$|\Omega|$	Deterministic Equivalent Model				Scenario-related Model			
		m	nc	$n01$	dens(%)	m	nc	$n01$	dens(%)
P1	7	2825	6125	22	0.098	425	875	22	0.66
P2	7	4415	12250	37	0.062	665	1750	37	0.42
P3	7	5006	15120	33	0.052	746	2160	33	0.35
P4	13	5103	11700	30	0.055	423	900	30	0.68
P5	13	3928	8125	25	0.072	328	625	25	0.88
P6	15	7228	18900	25	0.036	508	1260	25	0.53
P7	15	6025	13125	22	0.046	425	875	22	0.07
P8	51	49002	132192	39	0.005	1002	2592	39	0.27
P9	51	12772	25500	19	0.221	272	500	19	1.07
P10	51	20425	44625	22	0.014	425	875	22	0.07
P11	100	25022	50000	19	0.011	272	500	19	1.07
P12	150	37522	75000	19	0.007	272	500	19	1.07
P13	250	62522	125000	19	0.004	272	500	19	1.07
P14	5	40883	216000	80	0.005	8243	43200	80	0.027

Figure 1 shows the main results of the computational experimentation with our *BFC* approach. The legend is as follows: Z_{LP}, Z_{IP} and $\underline{Z}$ are the values of the *LP* solution, the stochastic solution and the initial feasible solution, respectively. Note: The values shown in the figure are scaled by dividing them by Z_{LP}, $Z_{IP} = 0$ for case P11, and the ratio $\underline{Z}/Z_{LP}$ does not appear in the cases P4, P6, P7, P9 and P11 due to $\underline{Z} = 0$, nor in the cases P8 and P14 due to the fact that OSL gives an infeasible solution as the *LP* optimal solution. Notice that the *BFC* approach obtains the optimal stochastic solution in all cases. On the other hand, observe that the approach for obtaining the initial solution is very efficient when it is feasible for all scenarios.
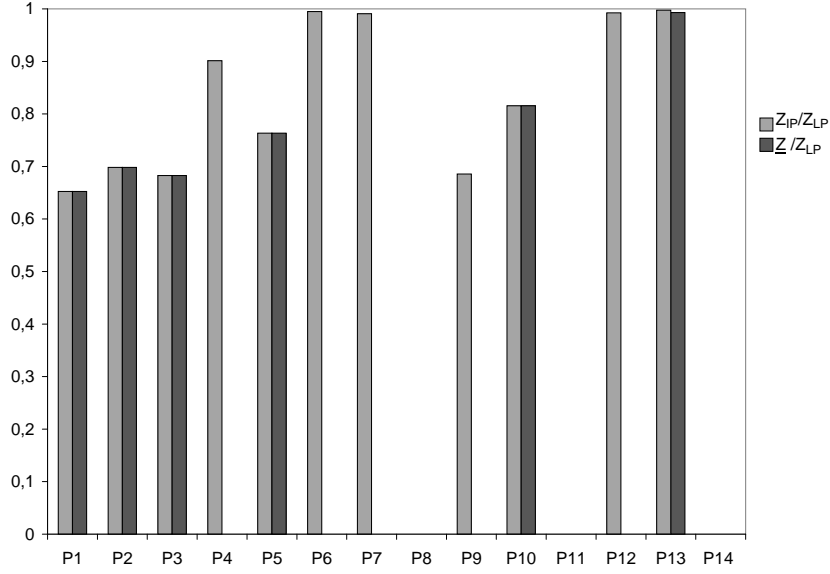
Figure 1: Ratios Z_{IP}/Z_{LP} and $\underline{Z}/Z_{LP}$

Table 3. Stochastic solution. Computational effort

| | OSL for DEM | | | | Proposed Algorithm | | | |
| | | | | | $\gamma - \delta$ bs | | $\delta - \gamma$ bs | |
Case	T_{LP}	T_{IP}	T	nn	nn	TT	nn	TT
P1	23.27	78.96	102.23	145	286	124.89	269	77.72
P2	92.68	475.40	568.98	288	128	193.09	108	151.61
P3	114.84	2127.03	2241.87	1088	1132	1582.17	694	1343.90
P4	80.05	160.63	240.68	64	546	270.63	72	42.88
P5	46.31	275.48	321.79	209	742	576.69	1882	843.70
P6	159.84	85.07	244.91	18	1076	1507.46	56	51.05
P7	107.19	94.15	201.34	27	1062	953.09	61	52.00
P8	>80000	-	-	-	1312	38913.94	421	8914.23
P9	1115.37	1946.23	3061.60	63	12	61.05	203	192.75
P10	4442.25	3864.41	8306.66	51	206	889.42	142	445.05
P11	3855.28	11901.81	15757.09	21	8	163.76	25	157.98
P12	13878.64	19206.31	33084.95	19	1842	4439.85	1774	3926.49
P13	39009.66	14384.75	53394.41	45	424	2262.75	1205	3938.49
P14	*	-	-	-	86	2159.69	241	1945.07

Note *: OSL gives an infeasible solution as the optimal one.

Table 3 shows the computational effort for obtaining the stochastic solution. The headings are as follows. *OSL for DEM*, plain use of the optimization engine to directly solve the model (2.1)–(2.16), compact representation; γ–δ bs (res., δ–γ bs), branch-and-fix strategy with priority given to the γ–variables (res., δ–variables) over the others (see the Appendix); T_{LP} and T_{IP}, computing time (secs.) to obtain the optimal LP solution and the optimal mixed 0–1 solution, respectively; $T = T_{LP} + T_{IP}$; TT, total time including the time required to obtain the initial feasible solution, the optimal LP solution and the optimal mixed 0–1 solution; and nn, number of branches in the branch-and-bound strategy. We can observe the increased efficiency of the proposed approach over the plain use of the optimization engine. Notice that the δ–γ branching strategy requires less computational effort than the γ–δ branching strategy by first branching on the plant dimensioning and then branching on the product selection. The plain use of

the optimization engine in case P13 requires almost 15 hours of computing time, while our approach requires less than 40 minutes, including the time to obtain the initial solution. Case 8 is extremely difficult; we interrupted the LP optimization with the plain use of OSL after 80000 secs. (more than 22 hours), but the proposed approach solves the problem in less than 11 hours. On the other hand, case P14 cannot be solved by plain use of OSL, but the proposed approach only requires 36 minutes to obtain the optimal mixed 0–1 solution.

Figure 2 shows some parameters for analyzing the goodness of the stochastic approach (see e.g. [5] for more details). The legend is as follows: Z_{IP}, solution value of the DEM (2.20); WS, *Wait-and-See* that can be expressed as $WS = \sum_{\omega \in \Omega} w^\omega Z_{IP}^\omega$, where Z_{IP}^ω is the solution value of model (3.1) for scenario ω; and EEV, *Expected* result of the *Expected Value* that can be expressed as $EEV = \sum_{\omega \in \Omega} w^\omega Z^\omega$, where Z^ω is the solution value of model (3.1) for scenario ω whose 0–1 variables have been set to the optimal solution of the model for the average scenario. We can observe that WS very frequently provides a better upper bound of the optimal stochastic solution than the bound Z_{LP} and, in any case, $EVPI$ is relatively small, where $EVPI = WS - Z_{IP}$ is the *Expected Value of Perfect Information*. On the other hand, we have that VSS is strictly positive for all cases, where $VSS = Z_{IP} - EEV$ is the *Value of the Stochastic Solution*. This gives a measure of the advantage of using stochastic programming approaches over scenario deterministic based schemes. Note: $Z_{IP}/WS = 0$ for case P11 and $EEV/WS = -\infty$ (infeasible) for case P14. The average scenario strategy gives solutions with losses in 4 out of the 14 cases, and an infeasible solution for case P14. The VSS is significatively high in 5 cases with positive benefit for both types of strategies. Particular attention should be given to the results for case P11; the stochastic solution does not recommend business, but the solution based on the average scenario does, incurring heavy losses.
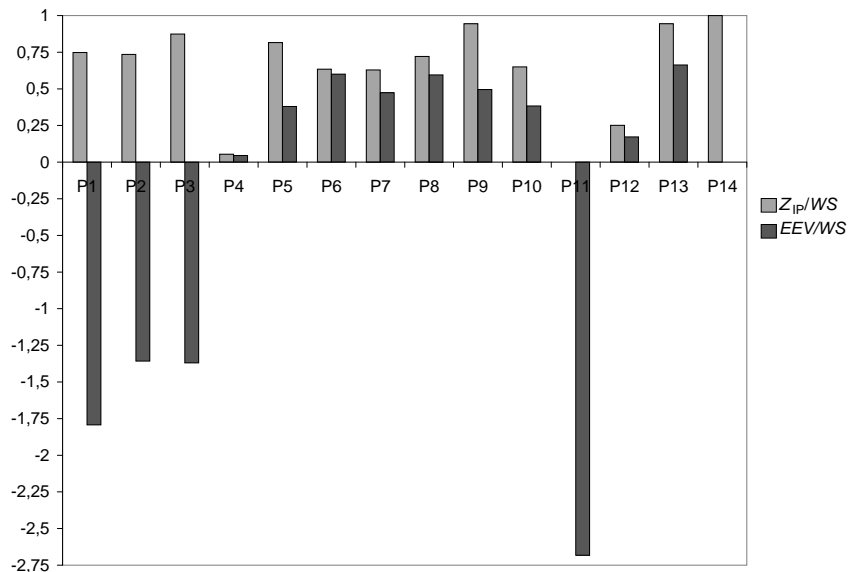

Figure 2: Ratios Z_{IP}/WS and EEV/WS

Figure 3 shows the probability that a scenario will occur with negative benefit for both the stochastic strategy and the average scenario strategy, based on the set of scenarios that are considered. For example, there is a probability of 0.833 that a scenario with losses will occur in case P2 for the average scenario strategy, but the probability is zero in case P11 for the stochastic strategy. Notice that the probability of incurring losses via the stochastic strategy is never greater than the related probability for the average scenario strategy, and it is smaller in 7 out of the 13 cases, are indeed much smaller in some of them.
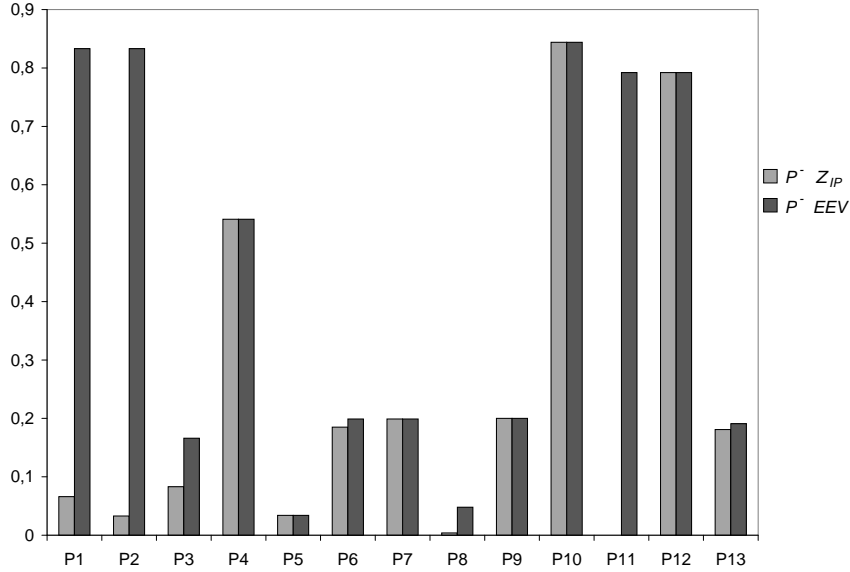
Figure 3: Probability of scenarios with negative benefit

5 Conclusions

Production planning is one of the most broadly studied fields of application in Deterministic Optimization. In this paper we present a strategic application case, where the treatment of uncertainty in product price, demand and production cost plays a central role in decision making. Different risk measures are considered, such as the *reaching probability* to maximize jointly with the (mean) expected functional, and the *Value-at-Risk* functional. The problem is represented by a mixed 0–1 *Deterministic Equivalent Model (DEM)* of a two-stage stochastic model with full recourse. Strategic decisions are represented by 0–1 variables. Tactical decisions are represented by continuous variables; they have nonzero elements only in the second stage constraints. The *DEM* is very large even for moderate case sizes. Its *splitting variable* representation via scenario allows us to convert the problem into a set of scenario related mixed 0–1 programs with *nonanticipativity constraints* for the first stage variables. A two-stage specialization of a *Branch-and-Fix Coordination* scheme, which we describe elsewhere, is presented to coordinate the execution of the branching and fixing phases for each scenario related program, such that the *nonanticipativity* constraints are satisfied and the mean objective function is optimized. For this purpose the concept of *Twin Node Families* is used. Moreover, the approach can be expanded straightforwardly to consider the mean-risk objective function as well. The proposed approach obtains the optimal solution in all cases of the testbed that we experiment with. The computational effort of the proposed approach favorably compares with the effort required to solve the *DEM*, compact version by plain use of the state-of-the-art optimization engine IBM OSL. The approach alse gives better results than the traditional average scenario based scheme in the expected objective function value and the probability of having a solution with negative benefit. Given the promising results obtained, it is planned to expand the proposed approach to the maximization of the *Value-at-Risk* functional.

Appendix

Different types of implementation can be considered within the algorithmic framework presented in section 3. This Appendix presents the two versions implemented to perform the

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computational experiment reported in Section 4.

There are three types of branching to perform, namely, branching on products, branching on plants and branching on plant dimensioning. We have chosen the *depth first* strategy for the *TNF* branching selection, first "branching on the zeros" and afterwards "branching on the ones" for the chosen 0–1 variable to satisfy the *nonanticipativity* constraints (2.21) for the selected *TNF* to branch. No branching on plants is performed until a full branching on products is completed (i.e., a set of products is selected), if the γ–δ branching strategy is used. Viceversa, no branching on products is performed until a full branching on plants (and the dimensions selected) is completed, if the δ–γ branching strategy is used. In any case, no branching on a given plant is performed until a full branching on the dimensioning of the preceding plant in the given branching order is completed (see below). Additionally, note that zero-branching on a given δ–variable, say, $\delta_i^{k\omega}$ implies that $\delta_i^{k\prime\omega} = 0$ for $k\prime = k + 1, \ldots, |K_i|$ and one-branching on the same δ–variable implies that one-branching has already been performed on the variables $\delta_i^{k\prime\omega}$ for $k\prime = 1, \ldots, k - 1$ (see constraints (2.4)).

Notice that a *TNF* can be pruned for either of the following two reasons: (a) the *LP* relaxation of the scenario-related model (3.1) attached to a node member is infeasible, and (b) there is no guarantee that a better solution than the *incumbent* one can be obtained from the best descendant integer *TNF* (in our current implementation, this is based on the *TNF* objective function value).

Once a *TNF* has been pruned, the branching criterion allows us to perform a "branching on the ones" (if the *TNF* has already been "branched on the zeros"). Otherwise, we *backtrack* to the previous branched *TNF*.

Notice that *backtracking* on plants must be done directly to the one-branching on the capacity (dimension) level whose δ–variable has been zero-branched for the plant, if any. If there is no such zero-branching (i.e., the full production capacity of the plant is considered in the current status of the branching process), the *backtracking* process must be deepened to the previous plant.

Another topic of interest is the branching order for products and plants. We consider a *static* order for both. The order for products is determined according to the non-increasing expected net income upper bound criterion, where the parameter, say, μ_j, $\forall j \in \mathcal{J}$, can be expressed as

$$\mu_j = \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}: j \in \mathcal{J}_i} \sum_{m \in \mathcal{M}_j} \sum_{t \in \mathcal{T}} w^\omega (b_{jt}^{m\omega} - d_{jt}^{im}) D_{jt}^{m\omega}. \tag{5.1}$$

The order for plants is determined according to the non-decreasing expected upper bound estimation criterion for the operation and investment depreciation costs, where the parameter, say, λ_i, $\forall i \in \mathcal{I}$, can be expressed as

$$\lambda_i = \beta_i \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}_i} \sum_{t \in \mathcal{T}} w^\omega c_{jt}^{i\omega} \overline{X}_{jt} + (1 - \beta_i) \sum_{k \in \mathcal{K}_i} a_i^k, \tag{5.2}$$

where $0 \leq \beta_i \leq 1$.

On the other hand, the branching order of the capacity levels for a given plant, say, i is the "natural" order $1, \ldots, |K_i|$.

Final note. Model (2.1)–(2.16) is preprocessed at each branching on the γ–,δ–variables. The preprocessing basically consists of zero-fixing the 0–1 variables and updating the lower and upper bounds of some constraints due to their implications in the model.

The two versions of the *BFC* algorithm to solve model (2.20) are based on the γ-δ and the δ-γ strategies, respectively. Add the following notation:

LP^ω, LP relaxation of the scenario-related model (3.1) attached to the current node from the BF tree $\mathcal{R}^\omega$ in the given TNF, for $\omega \in \Omega$.

Z_{LP}^ω, solution value of the LP model LP^ω for $\omega \in \Omega$. By convention, let $Z_{LP}^\omega = -\infty$ in case of infeasibility.

$\overline{Z}$, upper bound of the solution value of the original model (2.20) to be obtained from the best descendant integer TNF for a given family. It can be computed as $\overline{Z} = \sum_{\omega \in \Omega} w^\omega Z_{LP}^\omega$. Note: The value is reported as Z_{LP} when computed in Step 2 below.

Procedure 1: Two-stage BFC by using the γ-δ branching strategy

Step 0: Preprocess the instance.

Step 1: Obtain the initial solution for the problem (2.1)–(2.16) by using the procedure *HAFS*, see section 3. Its solution value $\underline{Z}$ is a lower bound of the optimal solution for the original problem. Set $Z_{IP} := \underline{Z}$.

Step 2: Solve the $|\Omega|$ independent LP models LP^ω, $\forall \omega \in \Omega$, and compute $\overline{Z}$.
If all γ– and δ–variables take 0–1 values and the *nonanticipativity* constraints (2.21) are satisfied, then the optimal solution for the original problem (2.20) has been found and, so, update $Z_{IP} := \overline{Z}$ and stop.

Step 3: Set $j := 1$ and go to Step 5.

Step 4: Reset $j := j + 1$.
If $j = |\mathcal{J}| + 1$ then go to Step 10.

Step 5: Branch $\gamma_j^\omega := 0$, $\forall \omega \in \Omega$.

Step 6: Solve the linear problems LP^ω, $\forall \omega \in \Omega$, and compute $\overline{Z}$.
If $\overline{Z} \leq Z_{IP}$ then go to Step 7.
If $\exists i \in \mathcal{I} \cup \mathcal{J}$ such that either the related δ–, γ–variables take different values for the $|\Omega|$ scenario-related models (3.1) or they take fractional values then go to Step 4.
Update $Z_{IP} := \overline{Z}$.

Step 7: Prune the branch.
If $\gamma_j^1 = 0$ then go to Step 9.

Step 8: Reset $j := j - 1$.
If $j = 0$ then stop, since the optimal solution has been found, being Z_{IP} the solution value.
If $\gamma_j^1 = 1$ then go to Step 8.

Step 9: Branch $\gamma_j^\omega := 1$, $\forall \omega \in \Omega$, and go to Step 6.

Step 10: Set $i := 1$ and go to Step 12.

Step 11: Reset $i := i + 1$.
If $i = |\mathcal{I}| + 1$ then go to Step 17.

Step 12: Set $k := 1$ and go to Step 14.

Step 13: Reset $k := k + 1$.
If $k = |K_i| + 1$ then go to Step 11.

Step 14: Branch $\delta_i^{k\omega} := 0$, $\forall \omega \in \Omega$.

Step 15: Solve the linear problems LP^ω, $\forall \omega \in \Omega$, and compute $\overline{Z}$.
If $\overline{Z} \leq Z_{IP}$ then go to Step 16.
If $\exists i \in \mathcal{I}$ such that either the related δ–variables take different values for the $|\Omega|$ scenario-related models (3.1) or they take fractional values then go to Step 13 provided that $\delta_i^{k1} = 1$ and, otherwise, go to Step 11.
Update $Z_{IP} := \overline{Z}$.

Step 16: Prune the branch.
If $\delta_i^{k1} = 0$ then go to Step 18.

Step 17: Reset $i := i - 1$.
If $i = 0$ then go to Step 8.
Set $k := arg\{\delta_i^{k'1}$ directly fixed to 0 by branching, $k\prime \in \mathcal{K}_i\}$, if any and, otherwise, go to Step 17.

Step 18: Branch $\delta_i^{k\omega} := 1$, $\forall \omega \in \Omega$, and go to Step 15.

Procedure 2: Two-stage BFC by using the δ-γ branching strategy

Steps 0-2: As for the γ-δ branching strategy.

Step 3: Set $i := 1$ and go to Step 5.

Step 4: Reset $i := i + 1$.
If $i = |\mathcal{I}| + 1$ then go to Step 12.

Step 5: Set $k := 1$ and go to Step 7.

Step 6: Reset $k := k + 1$.
If $k = |K_i| + 1$ then go to Step 4.

Step 7: Branch $\delta_i^{k\omega} := 0$, $\forall \omega \in \Omega$.

Step 8: Solve the linear problems LP^ω, $\forall \omega \in \Omega$, and compute $\overline{Z}$.
If $\overline{Z} \leq Z_{IP}$ then go to Step 9.
If $\exists i \in \mathcal{I} \cup \mathcal{J}$ such that either the related δ–, γ–variables take different values for the $|\Omega|$ scenario-related models (3.1) or they take fractional values then go to Step 6 provided that $\delta_i^{k1} = 1$ and, otherwise, go to Step 4.
Update $Z_{IP} := \overline{Z}$.

Step 9: Prune the branch.
If $\delta_i^{k1} = 0$ then go to Step 11.

Step 10: Reset $i := i - 1$.
If $i = 0$ then stop, since the optimal solution has been found, being Z_{IP} the solution value.
Set $k := arg\{\delta_i^{k'1}$ directly fixed to 0 by branching, $k\prime \in \mathcal{K}_i\}$, if any and, otherwise, go to Step 10.

Step 11: Branch $\delta_i^{k\omega} := 1$, $\forall \omega \in \Omega$, and go to Step 8.

Step 12: Set $j := 1$ and go to Step 14.

Step 13: Reset $j := j + 1$.
If $j = |\mathcal{J}| + 1$ then go to Step 17.

Step 14: Branch $\gamma_j^\omega := 0$, $\forall \omega \in \Omega$.

Step 15: Solve the linear problems LP^ω, $\forall \omega \in \Omega$, and compute $\overline{Z}$.
If $\overline{Z} \leq Z_{IP}$ then go to Step 16.
If $\exists j \in \mathcal{J}$ such that either the related γ–variable takes different values for the $|\Omega|$ scenario-related models (3.1) or it takes fractional values then go to Step 13.
Update $Z_{IP} := \overline{Z}$.

Step 16: Prune the branch.
If $\gamma_j^1 = 0$ then go to Step 18.

Step 17: Reset $j := j - 1$.
If $j = 0$ then go to Step 10.
If $\gamma_j^1 = 1$ then go to Step 17.

Step 18: Branch $\gamma_j^\omega := 1$, $\forall \omega \in \Omega$, and go to Step 15.

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